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MATHEMATICAL MODELING OF THE DISTRIBUTION OF CAPITAL INVESTMENTS IN ECONOMIC SECTORS

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Abstract

The article considers the model of the capital investment distribution plan, which takes into account various factors, and also defines criteria for its selection. The presented capital investment plan will be called the basic plan. In addition to the basic plan, other capital investment distribution plans are possible, bringing all sectors of the economy to a standard state. The authors solve the problem of harmonious development of the economic sector and maximum smoothing of existing imbalances. Variables for determining the quantitative state of economic sectors determined for a long planning period are considered. Economically, the task can be formulated as follows: it is necessary to obtain such a distribution of capital investments that would stimulate a reduction in the disproportion in the development of economic sectors. As a result, the task boils down to finding the optimal capital investment distribution plan among all acceptable capital investment distribution plans. The developed model of the capital investment distribution plan can be used in the real sector of the economy.

Keywords: Capital investments, distribution, economic sectors, model, plan

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1. Introduction

The economic meaning of the problem solved in the article is reduced to determining the harmonious development of economic sectors and the maximum smoothing of existing imbalances. As a result, the amount of funds needed to transfer all sectors of the economy from the initial state to the ideal state is determined, and the coefficient means the share of funds needed for the development of industries. After the distribution of capital investments by sectors of the economy in accordance with the plan, the quantitative state of the industries will change according to the formula presented by the authors. The formula is convenient for practical use, since it is based on the current state and automatically takes into account factors such as non-fulfillment of plans of previous years for individual industries, over-fulfillment of plans, etc. The capital investment plan determined by the formula is called the basic plan. In addition to the basic plan, other capital investment distribution plans are possible, bringing all industries to a standard state. A complete set of capital investment distribution plans is defined as a set of solutions to a system of equations and inequalities (Avila et al., 2019; Nemnyugin et al., 2020).

2. Problem Statement

We will denote by indices $\delta = 1,2,\ldots,n$ whether the economic sectors.

Let x_{δ_0} – be the initial quantitative state of industries $\underline{\underline{\delta}}$,

$\frac{1}{c_s}$ – industry growth rate when investing single capital init

A_t – total nutritional investment for all sectors of the economy of year t,

P_t – the number of population by the end long P canning period T.

R_s – regulatory efficiency of industries $\underline{\underline{\delta}}$.

x_{sT} – ideal quantitative state by the end of the long-term planning period, determined by Strategic Economic Development Plan.

v_{st} – investment invested in the industry Per year 1. It is assumed that the A sequence of numbers$A_0, A_1, \ldots, A_n$ (investment in years $0, 1, \ldots, T$) if P_t denotes the number of inhabitants by the end of the long-term planning period T and R_s – standard industry Coefficients, then the development goal of this industry is to fulfil the equality

$$X_{sT} = r_s P_T$$

3. Research Questions

To solve the problem, it is necessary to formalize the conditions of the problem, determine the amount of funds needed to transfer all sectors of the economy from the initial state to the ideal state, and a coefficient meaning the share of funds needed for the development of industries (Back, 2018). Then draw up a plan for the distribution of capital investments by sectors of the economy, taking into account changes in the quantitative state of industries according to the formula presented by the authors (Malafeyev et al., 2020b; Zaitseva et al., 2020b).

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4. Purpose of the Study

The main goal solved in the article is to determine the harmonious development of economic sectors and the maximum smoothing of existing imbalances using mathematical methods (Baicherova et al., 2020).

5. Research Methods

If the share of funds allocated to each branch does not change from year, then development from the initial state x_{δ_0} to give final state x_{sT} will occur only under the condition (Bondar et al., 2019).

For this industry is allocated in each year $\underline{\underline{t}}$ the amount

$$\overline{v}_{sT} = \frac{c_s(R_s P_T - X_{s0})}{\sum_{j=1}^{n} c_j(R_j P_T - x_{j0})} A_i; \qquad \delta = 1,2, \dots., n \tag{1}$$

Let's define the economic meaning of this expression. The number (1) contains the expressions:

$$c_s(R_s P_T - X_{s0})$$

Which is the sum of funds enquired to transfer. The industries $\underline{\underline{s}}$ from the initial state x_{s0} the ideal state

$$X_{sT} = R_s P_T$$

Denominator

$$\sum_{j=1}^{n} c_j(R_j P_T - x_{j0})$$

Represents the amount of funds required to transfer all branches of the economic sectors from state

$$x_{10}, x_{20}, \dots., x_{s0}, \dots, x_{n0}$$

In states

$$x_{1T}, x_{2T}, \dots, x_{sT}, \dots., x_{nT}$$

Thus, the coefficient at A, in expression a, means the share of funds needed for the development of industries. After the distribution of capital investments in accordance with the plan a in the year 0, the quantitative state of the industries will change according to formula

$$x_{s1} = x_{s0} + \frac{1}{c_s} \bar{v}_{s_0}, s = 1,2, \dots., n$$

If we reapply the distribution of capital investment according to plan a then the quantitative state of the industry at time I can be calculated by

$$x_{s,t+1} = x_{s_t} + \frac{1}{c_s} \bar{v}_{s_t}; t = 0,1, \dots, T-1. \tag{2}$$

Let's return to plan a, it easy to prove that the calculation of the values of u_{s_t} can also be carried out using as the initial value x_{s_0} the value x_{x_t} determined by formulas (2)

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$$\bar{v}_{s_t} = \frac{c_s(R_s P_T - x_{s_t})}{\sum_{j=1}^{n} c_j(R_j P_T - x_{j_t})} A(t). \quad (3)$$

Formula (3) is more convenient for practical use, science it is based on the current state x_{s_t} and, Thus, automatically take into account such factors as under fulfilment of the plans of previous years for individual industries, over fulfillment of plans, etc (Durakova et al., 2020).

The capital investment plan, determined by formula (3), we will call the baseline plan. Note that, in addition to the reference plan, other plans for the distribution of capital investment are possible leading all branches of the economic sectors to standard condition in particular, all plans of the form.

$$v_{s_t} = \bar{v}_{s_t} + \Delta_{s_t}, s = 1,2, \dots., n; t = 0,1, \dots, T$$

$$\Delta_{s_t} \geq -\bar{v}_{s_t}, s = 1,2, \dots, n, t = 0,1, \dots, T-1$$

$$\sum_{s=1}^{n} \Delta_{s_t} = 0, s = 1,2, \dots, n, t = 0,1, \dots, T-1 \quad (4)$$

Process the specified properties the values of v_{s_t} are obtained as variations of the base plan for the distribution of capital as variations of the base plan for the distribution of capital investments and indeed formulas (4)

$$\sum_{s=1}^{n} v_{s_t} = \sum_{s=1}^{n} c\, \bar{v}_{s_t} + \Delta_{s_t} = \sum_{s=1}^{n} \bar{v}_{s_t} + \sum_{s=1}^{n} \Delta_{s_t} = \sum_{s=1}^{n} \bar{v}_{s_t}$$

On the other hand, any plan for the distribution of capital investment can be describe by formulas (4). The entire set of plans for the distribution of capital investments in defined as the set of solution to the following system of equation and inequalities:

$$\sum_{s=1}^{n} v_{s_t} = A_t, t = 0,1, \dots, T-1 \quad (5)$$

$$v_{s_t} \geq 0, s = 1, \dots, n; t = 0,1, \dots, T-1$$

Here is a predetermined and known quantity. Obviously, the reference plan satisfies the condition (5). Let us return to the task of harmonious development of the economic sectors sector and the maximum smoothing of the existing disproportions set by us. The basic plan for the distribution of capital investment, to some extent, as can be seen from formula (4) already solves this task (Malafeyev et al., 2020a).

However, it’s possible, using some criterion of harmonies develop to solve the problem we will assume the state of the branches x_{s_T} is devoid of disproportions. Therefore, the degree of disproportionality of current quantitative state of the branches, although we will evaluate in relation to the state of the x_{s_T} will depend on. The distance of the current state from the normative state.

$$[x_{1t}, \dots, x_{st}, x_{nt}] \quad (6)$$

From the normative state

$$[x_{1T}, \dots, x_{sT}, \dots, x_{nT}] \quad (7)$$

Mathematicians to determine the “distance” between expansion (vectors) (6) b (7) suggest the following value

$$\sum_{s=1}^{n} r_s \left(\frac{x_{sT}-x_{st}}{x_{sT}}\right)^2, \quad \sum_{s=1}^{n} r_s - 1 \tag{8}$$

Expression (7) different from the usual Euclidean distance in denominators $\frac{x_{sT}}{\sqrt{y_s}}, s = 1,2,\dots,n$ which in our case, it makes sense to bring all industries to a single quantitative indicator.

Now we can define some criterion for choosing a capital investment allocation plan. Suppose that in year $\underline{t}$ we have allocated capital investment in accordance with plan satisfying condition (4). Then the "distance" between the new state of the industries $x_{s,t+1} = x_{st} + \frac{v_{st}}{c_s}, s = 1,2,\dots,n$ and normative, is determined by the formula (8) let us study expression (8) in more detail from point of view of the possibility of our influence on its value through the variables included in it. The variable of the hut, representing the ideal quantitative state of the branches of the economic sectors are determined for long planning period are considered. The state of the industries at the moment it are the results of the decision according to the distribution of capital investments adopted in the post (Bayer et al., 2020).

We will assume (This is, of course, a simplification) that the values of c_s also cannot be changed. Only v_{st} we believe that the planner can choose these values taking into account only constant (4). Any plan is called chalet capital plan$s = 1,2,\dots,n$.

Constants (4)

$$\sum_{s=1}^{n} v_{st} = A_t, t = 0,1,\dots,T-1$$

$v_{sT} \geq 0, s = 1,2,\dots,n; t = 0,1,\dots,T-1.$

Setting the task economically, we talked about the desire to obtain such a distribution of capital investments that would stimulate a decrease in the disproportion in the development of branches of the economic sectors.

As a measure of such disparities, we introduced criterion (8). That is, the problem was reduced to finding among all admissible plans for the distribution of capital investments of such a plan v_{st} which achieves min of expression (8) delivering min to expression (8) is called optimal plan for the distribution of capital investments (Zaitseva et al., 2020a).

Finally, the problem is mathematically formulated and solved as follows: in each current year t allocated investments according to the plan which is form the condition

$$min \sum_{s=1}^{n} v_s \left[\frac{x_{sT} - Cx_{st} + \frac{v_{st}}{c_s}}{x_{sT}}\right]^2$$

With restriction

$$\sum_{s=1}^{n} v_{st} = A_t; t = 0,1,\dots,T-1$$

$$v_{st} \geq 0, s = 1,2,\dots,n; t = 0,1,\dots,T-1$$

Algorithm for solving

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$$H(v) = \sum_{s=1}^{n} v_s \left[\frac{x_{sT} - (x_{st} + \frac{v_{st}}{c_s})}{x_{sT}} \right]$$

And $H(v)$ can be rewritten as

$$H(v) = \sum_{s=1}^{n} (a_s - c_s v_s)^2$$

Let us give an algorithm for solving this problem obviously the minimum point of the function $H(0)$ is if and only if

$$\max(-grad_v H, v - V^*) = 0$$

Or

$$max_{V \in M} (\sum_{i=1}^{n} c_i (a_i - c_i v_i)(v_i - v_i^*) =$$

$$max_{i=1,\ldots,n} \{c_i (a_i - c_i v^*)\} A - \sum_{i=1}^{n} c_i (a_i - c_i v_i^*) v_i = 0$$

$$\left(M - \left\{ V : \sum_{i=1}^{n} v_i = A;\ v_i \geq 0; i = 1,2, \ldots., n \right\} \right)$$

Hence if follows of is V^* minimum point if and only if when there a set $I \subset [1,2,3, \ldots, n]$

$$a_i c_i \geq \lambda, i \in I \qquad (9)$$

$$a_i c_i < \lambda; i \overline{\in} I$$

Were in

$$\lambda = \frac{\sum_{i \in I} \frac{a_i}{c_i} - A}{\sum_{i \in I} \frac{1}{c_i^2}}$$

$$V_i^* = \frac{a_i}{c_i} - \frac{\lambda}{c_i^2}, i \in I, v_i^* = 0, i \overline{\in} I$$

Is equivalent to inequality?

$$\frac{\sum_{i=1}^{k-1} \frac{a_i}{c_i} - A}{\sum_{i=1}^{k-1} \frac{1}{c_i^2}} \leq \frac{\sum_{i=1}^{n} \frac{a_i}{c_i} - A}{\sum_{i=1}^{R} \frac{1}{c_i^2}}$$

Is equivalent

$$a_k c_k \leq \left(\frac{\sum_{i=1}^{k-1} \frac{a_i}{c_i} - A}{\sum_{i=1}^{R-1} \frac{1}{c_i^2}} \right)$$

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This shows that the set – function H (A) does not decrease by inclusion, and for sufficiently large A, $I(A) = [1,2, \dots, n]$. It obviously follows from this that if you number the changes so that in the new numbering the in equalities

$$a_1 c_1 \geq a_2 c_2 \geq \cdots \geq a_n c_n$$

$$I = \{i: i \leq \delta\}$$

Where δ – the larget number at which the inequality n fulfilled

$$a_s c_s \geq \frac{\sum_{i=1}^{s} \frac{a_i}{c_i} - A}{\sum_{i=1}^{s} \frac{1}{c_i^2}}$$

Knowing set I, we can easily calculate by formulas

$$v_s^*, s = 1,2, \dots, n$$

In addition, we can recommend a simple approximation algorithm v_s^*, $s = 1,2, \dots, n$

$$v_s^* = \frac{a_s}{c_s} - \frac{1}{c_i} \frac{\sum_{i=1}^{n} \frac{a_i}{c_i} - A}{\sum_{i=1}^{n} \frac{1}{c_i^2}}, s = 1,2, \dots, n$$

6. Findings

Economically, the task was to obtain a distribution of capital investments that would stimulate a reduction in the disparity in the development of branches of the economy (Arzimbekov et al., 2020). As a measure of such differences, a criterion was introduced and the task was reduced to finding among all permissible capital investment distribution plans such a plan that reaches the minimum of expression (8) (Haber & Stornetta, 2017). It will be the optimal plan for the distribution of capital investments. The problem is mathematically formulated and solved by allocating investments in each current year in accordance with the plan and the specified conditions (Feller, 2017; Huang et al., 2020).

7. Conclusion

If may turn out that some of the meaning about calculated according to the above formula will be negative in this case, industries for which $v_s^* < 0$ must be excluded from consideration by setting the amount of capital investment to be placed in them equal to zero then, instead of the initial number of all industries in the initial prehab we solve the problem for a smaller number of industries we do this until all values of v_s^* become positive. This model can be complicated by introduction of additional linear constrains. For example, you can take into account the resource limit.

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