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THE RATIONAL USE OF RESOURCES PROVIDED TWO PRODUCTS OUTPUT, PART 2

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Abstract

The article is a component part of the research of using resources in the manufacturing. The research is carried out using the objective of the optimal resource exploitation. The manufacturing of two types of products is considered, where three resources are used. The main issue of the research is to define the conditions when all three resources will be completely consumed in the production process of two types of products. The relevance of the issue is to determine the production's external and internal conditions, when the resources are used rationally. The conditions are considered in relation to indexes of efficiency of production and resource reserves. We use the model's parameters compiled for the optimal resource exploitation issue, such as: the relative consumption of resources by product types, the relation of manufacturing efficiency indexes of two types of products, the relation of resource reserves. The check-out method of the rational resource exploitation is proposed for the optimal manufacturing of two types of products. The economic and mathematical model of resource exploitation in production process and linear programming methods are applied in the research. To find the optimal production plans, a simplex-method and Jordan-Gauss's double transformations are used. The article points out the connection between the rational resource exploitation and the issues of its replacement.

Keywords: Dual, linear programming problem, simplex-method

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1. Introduction

The issues related to the development of manufacturing require a wide range of materials to explore its effectiveness. These materials include techniques and methods from various fields of knowledge. Moreover, there are economic and mathematical methods among these very materials.

One of the conditions for effective manufacturing is the efficient resource exploitation and its rational consumption. The efficiency of resource exploitation is defined as a production when for a set of resources types R1, R2, …, Rm (m is the number of resources types), the index's collective value of the efficient manufacturing of products types A1, A2, …, An (n is the number of products types) is the largest (maximum). The rationality of resource exploitation is regarded as its manufacturing expenditure that for a set level of the index's total value, which determines the efficiency of production, the amount of resources of each type is minimal. Thus, a decrease in the amount of one of the resources causes a decrease in this very index's total value.

This implies the relevance of the rational exploitation research to determine the conditions to the production indexes when all resources are used rationally.

According to the article (Babin & Babina, 2021), which is the first part of this work, the problem of rational using of three resources in the manufacturing of two types of products is considered and set. There is a rational resource exploitation research for some conditions and we notice that there are other conditions for the production rationality. This article continues to explore the rationality of the production process.

The studies which are carried out in the article (Babin & Babina, 2021) are based on the materials of these very articles. In accordance with the article (Mamonov, 2016), the issues of production of n types of products using m types of resources are considered, where the problem of optimal resource exploitation is defined, presented in a linear programming form.

According to the article (Mamonov, 2016), for the optimal resource exploitation issue (or simply – the resource exploitation issue), an analysis of optimal solutions (production plans) for two types of products and two resources is carried out. According to the article (Mikhalchishina et al., 2020), the conditions for the manufacturing transition to a high level of automation for three technologies and two factors of production are applied: Labor and Capital. All issues are also led to a linear programming problem.

One of the linear programming founders is Leonid Vitalievich Kantorovich, a Soviet mathematician and economist. His works "Mathematical methods of organizing the production planning", (Kantorovich, 1939), and "Economic calculation of the best resource exploitation", (Kantorovich, 1960), are considered basic for linear programming. One of the main methods of linear programming is the simplex-method, which is an accurate method. It is widely used in linear programming. The developer of this method is considered to be George Bernard Danzig. One can highlight his works (Dantzig & Thapa, 1997, 2003).

This article proposes to take a review of the problem of optimal use of three resources in the manufacturing of two types of products and to determine the conditions when all three resources will be completely consumed in the optimal plan. The analysis of this very plan is examined by the simplex-method, which uses Jordan-Gauss's double step transformations (Mamonov, 2020).

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2. Problem Statement

In the article (Mamonov, 2016), the optimal resource exploitation issue is presented. In the first part of the research (Babin & Babina, 2021), this problem is formulated for three resources and two types of products. One can assume the problem of using three resources in the production of two types of products (Babin & Babina, 2021).

“The enterprise manufactures two types of products A1 and A2, using three resources R1, R2 and R3. The A1 unit of production requires a11 units of resource R1, a21 units of resource R2 and a31 units of resource R3, the A2 unit of production requires a12 units of resource R1 and a22 units of resource and a32 units of resource R3. Income from the sale of A1 unit of production is c1 rubles and A2 unit of production is c2 rubles. It is necessary to make a plan for the output of A1 and A2 products so the profit of the enterprise is maximal including the reserves of resources R1 in the amount of b1 units, R2 in the amount of b2 units, R3 in the amount of b3 units”.

Moreover, the objectives of the research are pointed out in the first part (Babin, Babina, 2021):

- to make an economic and mathematical model;

- to define the conditions for the income of each type of product and resource reserves, when all three resources are completely consumed according to the optimal production plan.

Also, the indexes of the relative resource expenditure for each type of resources are determined in the first part:

$$k_i = \frac{a_{i2}}{a_{i1}}, \quad (1)$$

where i = 1, 2, 3, and the ratio of production efficiency indexes A2 and A1:

$$k = \frac{c_2}{c_1}. \quad (2)$$

In the first part of the research (Babin & Babina, 2021), the conditions for the total resource expenditure are explored with the limitation:

$$k_1 < k < k_2. \quad (3)$$

In this article, we consider the values of the index k satisfying the two-sided inequality:

$$k_2 < k < k_3. \quad (4)$$

3. Research Questions

An economic and mathematical model of the problem is made (Babin & Babina, 2021); the following indexes of the problem are defined in the first part:

1) for the production plan:

x1 is the quantity of A1 products, x2 is the quantity of A2 products that the enterprise produces according to the plan;

2) supplementary variable of the direct problem yi, i=1, 2, 3;

3) supplementary variables of the dual problem vj, j=1, 2;

4) the production efficiency index Z in the direct problem, which is considered as income;

5) the production efficiency index W in the dual problem.

The very economic-mathematical model of the problem has the form:

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$$\begin{cases} a_{11}x_1 & +a_{21}x_2 & \le & b_1 \\ a_{21}x_1 & +a_{22}x_2 & \le & b_2 \\ a_{31}x_1 & +a_{32}x_2 & \le & b_3 \end{cases} . \qquad (5)$$
$$x_1 \ge 0 \quad x_2 \ge 0 \quad \ge \quad c_1$$
$$Z = c_1x_1 \quad +c_2x_2 \quad \to \quad \max$$

The dual problem is presented in the following form:

$$\begin{cases} a_{11}u_1 & +a_{21}u_2 & +a_{31}u_3 & \ge & c_1 \\ a_{12}u & +a_{22}u_2 & +a_{32}u_3 & \ge & c_2 \end{cases}$$
$$u_1 \ge 0 \quad u_2 \ge 0 \quad u_3 \ge 0 \quad \le \quad n_1 \ . \qquad (6)$$
$$W = b_1u_1 \quad +b_2u_2 \quad +b_3u_3 \quad \to \quad \min$$

Furthermore, the supporting indicators of the economic and mathematical model are determined:

$\beta_{is}^{(j)} = \frac{a_{sj}}{a_{ij}}$, (7)

where i=1, 2, 3;s=1, 2, 3;i≠s; j=1, 2;

$\beta_{is} = \frac{b_s}{b_i}$, (8)

where i=1, 2, 3; s=1, 2, 3; i≠s.

These indicators determine the relative resource expenditure for the unit of production A1 and the unit of production A2, formula (7), and the ratio of the corresponding resources reserves, formula (8).

As in the first part, we choose the order of resources so that:

$k_i \le k_s$, (9)

if i <j, and the following conditions are imposed on the resource expenditure unit of production A1:

$a_{11} \ne 0, a_{21} \ne 0, a_{31} \ne 0$. (10)

One can also assume in this work that:

$k_1 < k_2 < k_3$. (11)

We point out the conditions, when all three resources are used rationally if three types of products are manufactured. Thus, we consider the optimal plans for the products output, assuming more efficient resource exploitation. Rationality means that the resource is completely consumed according to the plan. This corresponds to the equality of its remainder to zero. The supplementary variables of the direct problem do determine the values of the remaining resources for the defined plan. Therefore, the values of supplementary variables are equal to zero according to the optimal plan with the rational resource exploitation:

$$y_1^* = 0, \quad y_2^* = 0, \quad y_3^* = 0. \qquad (12)$$

4. Purpose of the Study

We proceed to solving the linear programming problem (5) - (6). We solve it by the simplex-method, using Jordan-Gauss's double step transformations, which is considered by Mamonov (2020) in the articles. We note that the problem (5) is given in a standard form, so there is no need in its transformation. An initial simplex-table is presented in the first part of the research (Table 1). One can presume it. The u_i variables define the estimation of resources in the dual problem.

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Table 1. Initial simplex table of problems (5) - (6) (Babin & Babina, 2021)

0	$-x_1, v_1$	$-x_2, v_2$	1, W
y_1, u_1	a_{11}	a_{12}	b_1
y_2, u_2	a_{21}	a_{22}	b_2
y_3, u_3	a_{31}	a_{32}	b_3
Z, 1	$-c_1$	$-c_2$	0

4.1. The optimal plan findings

According to the article (Babin & Babina, 2021), table 1 corresponds to the admissible plan. Since $k_2<k<k_3$, the condition (4) is satisfied, we consider the main resources R_2 and R_2. Therefore, we choose the second and third rows as resolving lines for Jordan-Gauss's double transformation. The first and second columns are permissive, there are simply no other ones. We carry out Jordan-Gauss's double transformation. The results are presented in table 2. This very table is made for the first and second lines in the first part of the work.

Table 2. Simplex table of Jordan-Gauss's double step transformations

1	$-y_2, u_2$	$-y_3, u_3$	1, W
y_1, u_1	$-\frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$	$-\frac{\begin{vmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$	$\frac{\begin{vmatrix} a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \\ a_{11} & a_{12} & b_1 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$
x_1, v_1	$\frac{a_{32}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$	$-\frac{a_{22}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$	$\frac{\begin{vmatrix} b_2 & a_{22} \\ b_3 & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$
x_2, v_2	$-\frac{a_{31}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$	$\frac{a_{21}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$	$\frac{\begin{vmatrix} a_{21} & b_2 \\ a_{31} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$
Z, 1	$\frac{\begin{vmatrix} c_1 & c_2 \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$	$\frac{\begin{vmatrix} a_{21} & a_{22} \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$	$-\frac{\begin{vmatrix} a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \\ c_1 & c_2 & 0 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$

We check the optimality of the 2nd table's plan. One can consider the elements of the single column and Z -row (we do not consider the Z-element), which we denote with the upper index 2. Firstly, we examine the elements of the single column:

$$b_2^{(2)} = \frac{\begin{vmatrix} b_2 & a_{22} \\ b_3 & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}} = \frac{b_2 \cdot \left(\beta_{23}^{(2)} - \beta_{23}\right)}{a_{21} \cdot \left(\beta_{23}^{(2)} - \beta_{23}^{(1)}\right)} > 0, \qquad (13)$$

$$b_3^{(2)} = \frac{\begin{vmatrix} a_{21} & b_2 \\ a_{31} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}} = \frac{b_2 \cdot \left(\beta_{23} - \beta_{23}^{(1)}\right)}{a_{22} \cdot \left(\beta_{23}^{(2)} - \beta_{23}^{(1)}\right)} > 0. \qquad (14)$$

The admissibility of the 2nd table's plan requires both elements to be strictly greater than zero. Therefore, the condition is fulfilled:

$$\beta_{23}^{(1)} < \beta_{23} < \beta_{23}^{(2)}. \ (15)$$

Then we consider the elements of the Z-line:

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$$c_1^{(2)} = \frac{\begin{vmatrix} c_1 & c_2 \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}} = \frac{c_1 \cdot (k_3 - k)}{a_{21} \cdot (k_3 - k_2)} > 0, \quad (16)$$

$$c_2^{(2)} = \frac{\begin{vmatrix} a_{21} & a_{22} \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}} = \frac{c_1 \cdot (k - k_2)}{a_{31} \cdot (k_3 - k_2)} > 0. \quad (17)$$

Since $k_2<k<k_3$, both elements are strictly positive. It is enough for the optimal plan:

$$b_1^{(2)} = \frac{\begin{vmatrix} a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \\ a_{11} & a_{12} & b_1 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{22} \end{vmatrix}} \geq 0. \quad (18)$$

The total expenditure of the resource R_2 means that the element $b_3^{(2)}$ is equal to zero. Then the condition must fulfill:

$$\begin{vmatrix} a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \\ a_{11} & a_{12} & b_1 \end{vmatrix} = 0. \quad (19)$$

Thus, the conditions (4), (15) and (19) must be satisfied in order to use all three resources according to the optimal plan.

One can note that the resource R_2 is excess due to $\beta_{23} < \beta_{23}^{(1)}$, and the resource R_3 is excess due to $\beta_{23} > \beta_{23}^{(2)}$. We can make these very conclusions by referring to the work, which presents a table of solutions for the problem of using two resources in the manufacturing of two types of products.

According to table 2, it is an optimal direct problem plan:

$$X_1^* = \left(\frac{b_2 \cdot \left(\beta_{23}^{(2)} - \beta_{23}\right)}{a_{21} \cdot \left(\beta_{23}^{(2)} - \beta_{23}^{(1)}\right)}; \frac{b_2 \cdot \left(\beta_{23} - \beta_{23}^{(1)}\right)}{a_{22} \cdot \left(\beta_{23}^{(2)} - \beta_{23}^{(1)}\right)} \right). \quad (20)$$

According to table 2, we determine the optimal resource estimates in the dual problem:

$$U_1^* = \left(0; \frac{c_1 \cdot (k_3 - k)}{a_{21} \cdot (k_3 - k_2)}; \frac{c_1 \cdot (k - k_2)}{a_{31} \cdot (k_3 - k_2)}\right). \quad (21)$$

The optimal solution is not the only one in the dual problem, because there is zero in the single column in the first row, table 2. We consider the question of the existence of a second optimal solution in the dual problem.

In a simplex table, table 2, we choose the first line as the resolving one, since there is zero in the optimal plan.

We determine the maximum dual simplex-ratio of the columns for the first row. The maximum is the dual simplex-ratio of the first column, since the element $a_{12}^{(2)}$ is strictly less than zero and the element $a_{11}^{(2)}$ is strictly greater than zero, if the double inequalities (4) and (11) are satisfied. We choose the first column as permissive. The permissive element is $a_{11}^{(2)}$.

4.2. Transition to the second optimal plan

We move to the second optimal plan in the dual problem. We transform table 2 using modified Jordan-Gauss's transformations. We mark the elements with an upper index (3) in the new simplex-table. Since we are to get the optimal plan again, we search for new values of the elements of the single column and *Z*-row.

We note that the elements values of the single column do not change, because it contains zero in the resolving (third) row. We need to recalculate only the elements of the *Z*-row.

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$$c_1^{(3)} = -\frac{\frac{\begin{vmatrix} c_1 & c_2 \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}}{-\frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}} = \frac{\begin{vmatrix} c_1 & c_2 \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}} = \frac{a_{31} \cdot c_1 \cdot (k_3 - k)}{a_{11} \cdot a_{31} \cdot (k_3 - k_1)} = \frac{c_1 \cdot (k_3 - k)}{a_{11} \cdot (k_3 - k_1)}. \quad (22)$$

$$c_2^{(3)} = \frac{c_2^{(2)} \cdot a_{11}^{(2)} - c_1^{(2)} \cdot a_{12}^{(2)}}{a_{12}^{(2)}} = \frac{-\frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}} \cdot \frac{\begin{vmatrix} a_{21} & a_{22} \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}} + \frac{\begin{vmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}} \cdot \frac{\begin{vmatrix} c_1 & c_2 \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}}{-\frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}}. \quad (23)$$

According to the calculations, the element $c_2^{(3)}$ is figured out by the formula:

$$c_2^{(3)} = \frac{\begin{vmatrix} a_{11} & a_{12} \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}} = \frac{c_1 \cdot (k - k_1)}{a_{31} \cdot (k_3 - k_1)}, \quad (24)$$

$$Z^{(3)} = -\frac{\begin{vmatrix} a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \\ c_1 & c_2 & 0 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}. \quad (25)$$

As a result, we get a simplex table (table 3).

Table 3. Simplex table of Jordan-Gauss's double step transformations

3	$-y_1, u_1$	$-y_3, u_3$	1, W
y_2, u_2			0
x_1, v_1			$\frac{\begin{vmatrix} b_2 & a_{22} \\ b_3 & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$
x_3, v_3			$\frac{\begin{vmatrix} a_{21} & b_2 \\ a_{31} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$
Z, 1	$\frac{\begin{vmatrix} c_1 & c_2 \\ a_{31} & a_{32} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}$	$\frac{\begin{vmatrix} a_{11} & a_{12} \\ c_1 & c_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}$	$-\frac{\begin{vmatrix} a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \\ c_1 & c_2 & 0 \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}}$

5. Research Methods

We verify the plans' optimality of the direct and dual problems corresponding to table 3.

After the 2nd table conversion, the elements of the single column do not change, since there is zero in the permissive row in the single column. The plan is admissible. We check the plan optimality; we consider the elements of the Z-row. According to the formulas (22) and (24) we have:

$$c_1^{(3)} = \frac{c_1 \cdot (k_3 - k)}{a_{11} \cdot (k_3 - k_1)}, \quad (26)$$

$$c_2^{(3)} = \frac{c_1 \cdot (k - k_1)}{a_{31} \cdot (k_3 - k_1)}. \quad (27)$$

The expressions (26) and (27) are strictly greater than zero if:

$$k_1 < k < k_3. \quad (28)$$

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The optimal plan of the direct problem does not change. The second optimal plan for the dual problem:

$$U_2^* = \left(\frac{c_1\cdot(k_3-k)}{a_{11}\cdot(k_3-k_1)}; 0; \frac{c_1\cdot(k-k_1)}{a_{31}\cdot(k_3-k_1)}\right),\ V_1^* = (0; 0). \quad (29)$$

The objective functions values of the dual problems pair also do not change and are equal (25).

One can define the optimal solutions in the dual problem as in the first part of the research, [1]. The plan U_1^* and the plan U_2^* are optimal and basic in the dual problem. Therefore, all optimal plans are determined from the equation:

$$U^* = U_1^* t_1 + U_2^* t_2, \quad (30)$$

where

$$t_1 \geq 0, t_2 \geq 0, \quad (31)$$

$$t_1 + t_2 = 1. \quad (32)$$

The optimal plans' multiplicity is determined by the expression:

$$U^* = \left((1-t_1)\cdot\frac{c_1\cdot(k_3-k)}{a_{11}\cdot(k_3-k_1)}\frac{c_1\cdot(k-k_1)}{a_{31}\cdot(k_3-k_1)}; t_1\cdot\frac{c_1\cdot(k_3-k)}{a_{21}\cdot(k_3-k_2)}; \frac{c_1}{a_{31}}\left(\frac{k-k_1}{k_3-k_1} - t_1\cdot\frac{k_2-k_1}{k_3-k_1}\cdot\frac{k_3-k}{k_3-k_2}\right)\right) \quad (33)$$

where

$$0 \leq t_1 \leq 1. \quad (34)$$

Thus, we have obtained the conditions for the optimal manufacturing of two types of products when all three resources are completely consumed if k2<k<k3. To make it happen for the reserves of resources b1, b2 and b3 the conditions (15) and (19) must be satisfied. The plan (20) is optimal in this case, and the limiting resource estimates are determined by formula (33) with the condition for parameter (34).

6. Findings

We compare the solutions and conditions obtained in part 1 of work (Babin & Babina, 2021) and in this work. The target plan for values k1<k<k2, k2<k<k3 is the same, but obtained in different ways.

In the first case, when k1<k<k2, we find the optimal plan for the resources R1 and R2 according to its reserves b1 and b2, and then we check the condition of equality of the reserve b3 of the resource R3 to its consumption according to the very plan.

In the second case, when k2<k<k3, one has already had the optimal plan for resources R2 and R3 and we check the equality condition of the reserve b1 of the resource R1 with the consumption according to the plan defined by the reserves b2 and b3.

In both cases, we have the same plan, because the condition (19) is satisfied.

If there is no equality, according to the optimal plan the resources are not used rationally, namely:

1) if the consumption is less than the reserve, the remaining resource is not completely used;

2) if the resource consumption is greater than the reserve, the plan obtained for the previous resources is not optimal.

We are to check the conditions for the index β12 or β23 in advance. The condition must be satisfied for k1<k<k2:

$$\beta_{12}^{(1)} < \beta_{12} < \beta_{12}^{(2)}, \quad (35)$$

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and it is the condition (15) for k2<k<k3. The violation of the conditions (15) and (35) means that one of the resources taken for the calculation is not in short supply. Thus, the resource R1 is initially redundant for k1<k<k2, and $\beta_{12} < \beta_{12}^{(1)}$; and the resource R2 is initially redundant for $\beta_{12} > \beta_{12}^{(2)}$. If k2<k<k3, the resource R2 is redundant for $\beta_{23} < \beta_{23}^{(1)}$, and the resource R3 is redundant for $\beta_{23} > \beta_{23}^{(2)}$ according to the optimal plan.

We denote that according to the optimal plan both types of products are manufactured in these very cases of the coefficient k values. If k<k1 or k>k3 only one type of product is made, A1 and A2, respectively, in accordance with the optimal plan. In these cases, the rational resource exploitation in optimal plans is achieved when the resource reserves are proportional to the specific resource expenditure in the preferable product's manufacturing.

We point out that in these cases, the condition (19) is also satisfied, because the last column is proportional to either the first or the second column. Therefore, the condition (19) is a necessary condition for the rational resource exploitation in the manufacturing of two types of products using three resources.

7. Conclusion

The rational consumption of three resources in the manufacturing of two types of products imposes conditions on the resource reserves. The resource reserves with the very consumption are linearly dependent and the reserve of one of the resource is equal to the minimum quantity, sufficient for the production plan, determined by the full consumption of the other two resources. We assume that both of these resources are in short supply. According to the condition $k_1 < k < k_2$, it is more expedient to focus on the consumption of resources R1 and R2, and according to the condition $k_1 < k < k_2$ one should focus on the consumption of resources R2 and R3.

We note that according to the condition $k < k_1$ we achieve the total resource expenditure by the proportionality of the resource reserves to its specific expenditure for the output of A1, and according to $k > k_3$ we achieve it by proportionality to the specific expenditure of A2 output. The cases $k = k_1$, $k = k_2$and $k = k_3$ are special and require separate study.

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