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METHODS FOR SOLVING MARKET PROBLEMS BY SCIENTISTS IN MEDIEVAL CENTRAL ASIA

Issin Meyram (a)*
*Corresponding author

(a) L.N. Gumilyov Eurasian National University, 2, Satbayev Str., Nur-Sultan, Kazakhstan, issin.meyram@gmail.com

Abstract

The development of mathematics in Central Asia is widely reflected in scientific research and popular science literature on the history of mathematics, and in periodicals, but there are no studies focused on the interaction between mathematical methods and market problems in medieval Central Asia. In order to reveal the evolution of the interaction between mathematical methods and market problems in medieval Central Asia and its conditions, historical analysis and theoretical analysis of philosophical, mathematical and methodological literature were employed in the study. The object of the study is the interaction between mathematical methods and market problems in the works by scientists of medieval Central Asia. The subject of the study is mathematical methods used to solve market problems. The periodization of the development of science in medieval Central Asia defined by the orientalist G. Sieter revealed the evolution of the interaction between mathematical methods and market problems and its conditions in medieval Central Asia. In the Middle Ages, the algebraic treatise by al-Khwarizmi Short book on the calculus of algebra and almukabala was a breakthrough scientific work in Central Asia. First, al-Khwarizmi, then al-Buzjani and al-Biruni used mathematical methods in their works to solve market problems. The results of the study reveal the evolution of the interaction between mathematical methods and market problems in medieval Central Asia and its conditions. The results are of relevance since they help fill the gap in the study of development of economic and mathematical methods in medieval Central Asia.

Keywords: Algebra, medieval Central Asia, mathematical methods, market problems, scientific works

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1. Introduction

There is now an extensive literature on the history of the development of mathematics in Central Asia. The necessary information is provided in a bio-bibliographic reference book (Matvievskaya & Rosenfeld, 1983) about mathematicians and astronomers of Muslim countries who lived in the 8th–17th centuries. The edition consists of three books. The first book includes an introductory paper and a reference section. The authors of the paper are Rosenfeld, Yushkevich, and Matvievskaya write: 'the purpose of this book is to provide bio-bibliographic information about mathematicians and astronomers of a vast region – Central Asia, Azerbaijan and the North Caucasus, Iran and Afghanistan, Turkey and North India, the Arab countries of Western Asia, North Africa and the Iberian Peninsula – over 8th–17th centuries with due regard to all available literature on the issue and data from the manuscripts' (Matvievskaya & Rosenfeld, 1983). The book states that the mathematicians of the Middle Ages al-Khwarizmi, al-Buzjani, al-Biruni, and al-Kashi used mathematical methods in their treatises to solve market problems. A market problem is defined as a problem of market relations, the solution of which requires the use of mathematical methods. The paper (Luther, 2018) describes the establishment of the Soviet school of the history of mathematical sciences in the medieval Near and Middle East. 'The Soviet school of the history of mathematical sciences in the medieval Near and Middle East was established and further developed by Yushkevich and Rosenfeld, the greatest historians of mathematics of the 20th century' (Luther, 2018). The study (Akhmedov, 1962), which dates back to the second half of the 20th century, emphasizes that the contribution of medieval scientists in Central Asia to mathematics has not been fully investigated. At present, the works of mathematicians of the Middle Ages in Central Asia, as well as studies into the natural-scientific work of scientists of that time, are being investigated (Komilov, 2000). A book was published about scientific discoveries, cultural achievements, progress and other events that took place in Central Asia (Starr, 2017).

The study explores the works of scientists of medieval Central Asia investigated by the scientists of the Golden Horde (Fazlyoglu, 2014; Fazlyoglu, 2015). Three papers by Fazlioglu are three parts of one work. These papers are devoted to the work *Masterpiece in computational science*, which appeared in the Golden Horde state and was transferred during the reign of Uzbek Khan (1313–1342) to the ruler of the Crimean ulus of the Golden Horde Abul-Muzaffer Giyaseddin Tuluktemir Bey. The author of the treatise is unknown. The work allows conclusion that representation of mathematical issues adheres to the tradition of al-Khwarizmi, al-Kashi and other mathematicians of medieval Central Asia, but the treatise does not consider the interaction between mathematical methods and market problems.

2. Problem Statement

This paper deals with the interaction between mathematical methods and market problems in medieval Central Asia, which has not been considered for a long time. First, this is due to the fact that historians of mathematics studied the development and teaching of arithmetic, algebra and other mathematical disciplines, without any focus on their interaction with market problems, and second, the works by Central Asian scientists of the Middle Ages, which consider market problems solved by mathematical methods, began to be translated much later. For example, the second part of the *Testament*

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book of the algebraic treatise (al-Khwarizmi, 1983) was absent in medieval Latin translations. 'This issue has not been covered for a long time in the historical and scientific literature. It was believed that from a mathematical point of view, it is not of interest, and the restrictions set in the condition were considered arbitrary. For the first time the *Testament book* was investigated in 1917 by Rushka, and then by Vileitner and Gandz' (al-Khwarizmi, 1983).

3. Research Questions

The object of the study is mathematical methods used to solve market problems. To solve market problems, scientists of medieval Central Asia employed the 'triple rule' method, the 'rule of five quantities' method, a linear equation, and a geometric progression. The *Testament book* by al-Khwarizmi contains more problems on the application of a linear equation, a number of problems on the application of an indefinite equation and one problem on the application of a system of two linear equations with two unknowns.

4. Purpose of the Study

The purpose of the study was to find out the conditions and evolution of the interaction between mathematical methods and market problems in the works by scientists of medieval Central Asia. To achieve the purpose, general scientific and special methods were used, including the historical method. The periodization of the development of science in medieval Central Asia was determined by the well-known historian of science and orientalist Ziter. He identified three periods: 1) from 750 to 900, 2) from 900 to 1275, and 3) from 1275 to 1600. The second period is divided into two sub-periods: a) from 900 to 1100, and b) from 1100 to 1275. (Komilov, 2000). The first period was called by him the period of translation activity and the development of the scientific heritage of the ancient Greek and ancient Indian peoples. Zuter claims that during the first sub-period (900–1100) science developed at the courts of the Arab Caliphs, Samanids, Ghaznavids, Bunds, Fatimids, Seljukids, etc. The second sub-period (1100–1275) is associated with the Maragha observatory and the scientific works by Nasir ad-Din at-Tusi (1201–1274) and his students and followers. The third period is mainly associated with the works of the Samarkand scientific school (Komilov, 2000).

5. Research Methods

The study employed narrative, historical, systemic and comparative methods. The narrative method was used to describe the interaction between mathematical methods and market problems in the works by scientists of medieval Central Asia based on the following: chronological scale, personalities, scientific works. The historical method based on Suther's periodization considers the evolution of the interaction between mathematical methods and market problems over three periods. Systemic and comparative methods were used to clarify the conditions for this interaction. The analysis of philosophical, mathematical literature and literature on the history of mathematics was carried out.

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6. Findings

The interaction between mathematical methods and market problems in Central Asia in 750–900.

Muhammad ibn Musa al-Khwarizmi (783–850) was the leading mathematician and astronomer among scientists in Baghdad and was the first to compose his work in Arabic (Baki, 1992) and used mathematical methods to solve market problems. His was born in the Khwarizm state. During the Middle Ages, Khwarizm was a 'powerful economic and cultural center in the East' (al-Khwarizmi, 1983). This is evidenced by materials from archaeological excavations and written sources. The Silk Road from China passed through Khwarizm. In addition, Khwarizm was engaged in lively trade with India and Iran at that time. Scientists from these states moved to Khwarizm together with trade caravans. In turn, scientists from Khwarizm visited other countries. This involved the exchange of scientific ideas between scientists. Due to economic ties, scientists of the trading states exchanged works on philosophy, mathematics, astronomy, and medicine. In these conditions, the formation of al-Khwarizmi as a scientist took place. According to Matviyevskaya, a scientist al-Khwarizmi worked under the patronage of al-Mamun when he moved to the city of Merv, which in the Middle Ages was 'the largest center of economic and cultural life' in Central Asia. Al-Mamun was the governor of the caliph, and then became the caliph in 813. He lived in Merv from 809 to 818. Later, Caliph al-Mamun moved to Baghdad with a group of Central Asian scientists, including al-Khwarizmi. With al-Mamun, science continued to develop in Baghdad, the capital of the Caliphate. The House of Wisdom was set up, which turned into the center of a large scientific school. Outstanding scientists from various areas of the Caliphate were brought to Baghdad. Due to his scholarship, al-Khwarizmi occupied a prominent position in the House of Wisdom.

In Baghdad, the scientific works of Greek and Indian scientists were translated into Arabic. These scientific works were brought to Baghdad not only through trade, but international relations also facilitated the exchange of scientific ideas.

The algebraic treatise *Short book on the calculus of algebra and almukabala* consists of two parts. The first part of the algebraic treatise by al-Khwarizmi contains the *Chapter on transactions* dedicated to the triple rule. Al-Khwarizmi notes that human transactions 'deal with four numbers set by the questioner – measure, price, quantity and value. The number equal to the measure stands against the number equal to the value, and the number equal to the price stands against the number equal to the quantity. The rule states that among three known numbers there are necessarily two, each of which stands against the other. Multiply each of the two known numbers standing against each other by the other, and divide the product by another known number standing against the unknown. If you have this quotient, it is an unknown number that the questioner asks about, it stands against the number which you divided by' (al-Khwarizmi, 1983). As an example, al-Khwarizmi offers a market problem and explains how to solve it: 'If the questioner asks: an employee whose monthly salary is ten dirhams worked six days, what is his share, then you know that six days is one fifth of the month and that his share of dirhams is the same as his share of the working time. The rule is as follows: a month that includes thirty days is a measure, ten dirhams is a price, six days is a quantity and [it is asked] what is the share, that is a cost; multiply the price (ten) by the amount that stands against it (six), the product is sixty, then divide it by thirty (known number), that is a measure, and you get two dirhams, that is a value' (al-Khwarizmi, 1983).

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The second part of the algebraic treatise considers many problems about the division of inheritance in accordance with the norms of Islamic law. The *Testament book* is also interesting in terms of market relations. At the same time, solving such problems, al- Khwarizmi operates with the concepts of the market: property, debt, price, redemption, monetary unit (dirhem), remuneration, etc.

In the book *A word on the classification of sciences*, the medieval thinker al-Farabi (870–950), who was the 'Second Teacher' after Aristotle in the East, reveals the meaning of 'the science of skillful methods.' As an example of the science of skillful methods, he cites the science called 'aljabr wa muqabala', that is, algebra, in which, for the first time in the history of science, the great mathematician al-Khwarizmi included not only a systematic presentation of algebra, but also methods and techniques of using mathematics in trade and in transactions of people with each other, or science, which contains the methods of practical arithmetic. This is evidenced by the following: 'Practical arithmetic (‘ilm al-‘adad al-‘amali), according to al-Farabi, studies the number of specific items to be counted and is used in commercial and civil cases. Civil cases (mu’amalat madaniyah) include problems of division of inheritance. Thus, the calculation of inheritance shares using al-Farabi's arithmetic is considered as a branch of mathematics' (Luther, 2019). The first page of al-Khwarizmi's manuscript states: 'This is the first book written about algebra and almukabal in (countries) of Islam' (al-Khwarizmi, 1983). The same is written in the notes to the treatise of al-Khwarizmi: 'A large volume of the *Testament book* indicates the importance that al-Khwarizmi attached to these problems. Since earlier Arabic sources are unknown, he was likely the first to apply algebraic rules to the solution of these problems' (al-Khwarizmi, 1983). Algebraic treatise by al-Khwarizmi is considered as a breakthrough work: 'Technologies of irrigation management, agriculture based on seasonal fluctuations in the amount of water, paper work in a complex tax system and ensuring the stability of the national currency – all this required knowledge of mathematics and technology. The followers of al-Khwarizmi developed these fields, especially mathematics and astronomy, to an astonishingly high level. It can be seen that al-Khwarizmi proceeded from local traditions in these fields when writing his breakthrough work' (Starr, 2017).

The use of mathematical methods in solving market problems in Central Asia in 900–1275.

An outstanding scientist and teacher Abu-l-Wafa al-Buzjani (940–998) (Hashemipour, 2007) worked under the patronage of the Caliph in Baghdad. After the death of the caliph, he was invited by the Khwarizmshah to the Khwarizm state. In new conditions, Abu-l-Wafa al-Buzjani continued to apply mathematical methods proposed by al-Khwarizmi in solving market problems and wrote a work devoted to methods for calculating land taxes and monetary investments. It was one of his books on mathematics written at the time (Starr, 2017). Medovoy studied the first part of the treatise and, characterizing the second (applied) part of the treatise, notes that it deals with the issues of 'charging for work, construction calculations, exchange and purchase of various types of grain, etc.'

Al-Buzjani was the mentor of Abu Nasr Mansur Ibn Iraq (960–1036), the heir to the ruling house of Khwarizm, who later became a talented astronomer, mathematician, and expert in trigonometry.

A famous scientist-encyclopedist of Central Asia Muhammad al-Biruni (973–1048) (Farmonova & Sultanov, 2020) was born in a wealthy family in Kyat, the capital of Khwarizm (now the city of Beruni in Uzbekistan). He was left without parents early. Abu Nasr Mansur Ibn Iraq, who knew al-Biruni's parents well, adopted him and raised him together with his son Ibn Iraq. Al-Biruni has a penchant for

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mathematics and astronomy, and, like Ibn Iraq, studied these fields. He studied and worked in the large scientific center Kyat. The Afghan Sultan Mahmud, who occupied Khwarizm in 1017, transferred him to the capital Ghazni, where al-Biruni headed a group of scientists gathered by Mahmud from the conquered countries. For several years al-Biruni lived in North India conquered by the Sultan, where he deeply studied scientific works in Sanskrit. Al-Biruni writes his *Work on mathematics, useful for merchants and government officials*, which is similar to the treatises by al-Khwarizmi and al-Buzjani. He is the author of the following works: *Book of enlightenment to the beginnings of the science of the stars, Monuments of past generations, Book of Indian rashiks* (Rosenfeld et al., 1973).

Rosenfeld, Rozhanskaya, and Sokolovskaya write about the works by al-Biruni: 'A special treatise, *Book of Indian rashiks* (Rashiki), Biruni dedicated to one of the methods of practical arithmetic widely used in the Middle and Near East and later in Europe – the 'triple rule'. ... Brahmagupta generalized this rule to 5, 7, 9 and 11 quantities that require the combined application of respectively 2, 3, 4 and 5 triple rules. Beruni explains in detail the direct and inverse rules and generalizes them to any odd number of quantities, giving tasks for 13, 15, 17 quantities' (Rosenfeld et al., 1973).

Consider the problem proposed by al-Biruni, which he solves using the 'rule of five quantities': 'If 10 dirhams bring income 5 dirhams in 2 months, how much income will 8 dirhams bring in 3 months?' To solve this problem, the quantities are placed according to the scheme.

10	8
2	3
5	x

Al-Biruni explains the 'rule of five quantities' as follows: 'To determine the unknown, five is transferred to an empty space, multiplied by 3, and then the product is multiplied by 8; it gives 120; it is remembered. Next, multiply 2 by 10, it gives twenty. What is remembered is divided by 20, in the quotient it will be 6; this is the income of 8 dirhams in 3 months.' The formula of the 'rule of five' is as follows:

$$x = \frac{5*3*8}{2*10} = 6$$

The 'chessboard problem' associated with an ancient Indian legend can be considered as practical content: it is required to find the total number of wheat grains if 1 grain is placed on the first field of the board, 2 on the second, 4 on the third, etc., doubling the number of grains in each next field (Rosenfeld et al., 1973). To solve this problem, al-Biruni uses a geometric progression.

Mathematical methods for solving market problems in Central Asia in 1275–1600.

The major scientific center of the medieval East was Samarkand. Timur's grandson, Ulugbek (1394–1449), the ruler, was seriously engaged in astronomy and mathematics. Ulugbek set up a scientific center as part of the Samarkand observatory. The observatory building was built in 1420. The scientist Dzhemshid al-Kashi (1380–1429) came to Samarkand from Iran. Djemshid al-Kashi headed the

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observatory in the first years (1420–1429). In his work *Key to arithmetic* (1427), he outlined methods for extracting roots using a formula (expressed verbally), which was later called Newton's binomial; proposed the approximate solution of the equation of the third degree and in the *Treatise on the circle* (1427) calculated π with 17 correct decimal places. 'In mathematics, he continued the long work initiated by al-Khwarizmi to introduce the decimal system and provided a systematic method for calculating decimal fractions' (Starr, 2017). The method for calculation with decimal fractions was used to solve market problems. The book by Bavrina and Fribus *Ancient Tasks* includes the problem of Jemshid al-Kashi on the application of the triple rule: 'The salary of an employee per month (30 days) is 10 dinars and a dress. He worked for 3 days and earned a dress. What is the cost of a dress?' To solve it, compose the proportion according to al-Khwarizmi.

30 days – x
10 days + x – 3,

where x is the cost of a dress, and, applying the rule formulated by al-Khwarizmi, we obtain an equation with one unknown:

$$\frac{(10+x)*3}{30}=x$$

Solve the equation and obtain the result that the cost of the dress is (1 + 1/9) dinars.

7. Conclusion

Thus, the Zuther's periodization yields the conclusion that the algebraic treatise by al-Khwarizmi *Short book on the calculus of algebra and almukabala* (750–900) for the first time in medieval Central Asia considers mathematical methods for solving market problems. After al-Khwarizmi, the interaction between mathematical methods and market problems continued due to scientific continuity in the works by al-Buzjani and al-Biruni in the period from 900 to 1275. The 'triple rule' method is developed in the works by al-Biruni; he generalizes this rule to any odd number of quantities, giving tasks for 13, 15, 17 quantities. In the 18th–19th centuries, the works by al-Khwarizmi, al-Buzjani, al-Biruni, and al-Kashi were not ignored, since at that time 'mathematics in madrasahs was taught using a certain system based on the works by Central Asian mathematicians. For example, Khwarizmi, Tusi, Kashi and others' (Akhmedov, 1962). 'In the last part of mathematics, complicated problems are solved on the distribution of property, according to Muslim traditions, between the heirs, which requires knowledge of mathematics' (Akhmedov, 1962).

In medieval Central Asia, the conditions for the evolutionary interaction between mathematical methods and market problems were as follows: lively trade of Central Asia with China, India, Iran; al-Khwarizmi, al-Buzjani, al-Biruni, al-Kashi were under the patronage of the caliphs and their wealthy confidants; mathematicians worked in the cities (scientific centers) of Kyat, Merv, Baghdad, Samarkand,

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where they studied mathematics, were engaged in mathematical research and translation of Greek and Indian works in mathematics.

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