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SHORT-TERM FORECASTING OF GROSS REGIONAL PRODUCT USING PRODUCTION FUNCTIONS

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Abstract

The article discusses the issues of short-term forecasting of gross regional product values, which is a key indicator of the functioning and development of regional socio-economic systems using the apparatus of production functions. Neoclassical two-factor neoclassical production functions with constant (CES - function) and variable (VES - function) elasticity of substitution of production cost factors of the system under consideration were used as tools. The economic indicators of the average number of people employed in the economy of the regional socio-economic system and the cost of its fixed assets, taking into account the speed of their retirement at the end of the year, were used, as factors of production costs. To build the above production functions, the index values of these indicators were used. The construction was carried out by the methods of mathematical statistics with the subsequent assessment of accuracy and adequacy by the corresponding generally accepted statistical criteria. After the forecast of changes in the values of indicators characterizing the factors of production costs, the latter were substituted into the constructed production functions to obtain a predictive estimate of the regional gross product. By the example of the economic system of the Khanty-Mansiysk Autonomous Okrug - Yugra, estimates were made of the changes in the values of this economic indicator with a forecast horizon of one year. The forecasting results were compared with the data of socio-economic statistics presented by the Federal State Statistics Service (Rosstat) in a regional context.

Keywords: Production function, regional economic system

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1. Introduction

Gross Regional Product (GRP) is a key economic indicator that characterizes the functioning and development (Alferova & Tretyakova, 2018; Drohobytsky, 2004; Kolemaev, 2005; Kleiner, 2012; Sokol et al., 2017) of the regional socio-economic system (RSES). To assess the value of GRP in modeling the functioning of RSES, the apparatus of production functions (PF) is often used (Alferova & Tretyakova, 2018; Drohobytsky, 2004; Kolemaev, 2005; Kleiner, 2012; Sokol & Kutyshkin, 2016; Sokol et al., 2017). The identified production functions, as a rule, are used to analyze the efficiency and effectiveness of spending aggregated factors of labor and capital costs in RSES (Alferova & Tretyakova, 2018). At the same time, the PF can also be considered as a tool for short-term forecasting of GRP RSES values. The main types of PFs used to solve the above problems are two-factor neoclassical production functions of the CES type — a function (CES PF), including the PF Cobb-Douglas (CD PF) (Alferova & Tretyakova, 2018; Drohobytsky, 2004; Kleiner, 2012; Sokol et al., 2017). Production functions of the type VES-function (VES PF) are used much less frequently (Sokol & Kutyshkin, 2016; Sokol et al., 2017).

2. Problem Statement

For short-term forecasting of changes in GRP RSES KhMAO - Yugra, we assume that the functioning of its economy is characterized by the following assumptions:

1. The functioning of the economy of this system can be described by two-factor neoclassical production functions and changes little over time, including for the adopted forecasting horizon.

2. The costs of the labor factor in the considered time interval change with a constant growth rate.

3. The lag between investments in fixed assets and the value of fixed assets of the regional economy is not taken into account.

4. The coefficient characterizing the disposal of fixed assets is considered known.

5. The economy of the region is considered closed, i.e. the direct influence of foreign trade on it is not directly taken into account.

3. Research Questions

Two-factor linearly homogeneous neoclassical PFs that are used to describe the functioning of the economy of the RSES of the RSES KhMAO - Yugra are in (Alferova & Tretyakova, 2018; Drohobytsky, 2004; Kleiner, 2012; Sokol et al., 2017):

- Constant elasticity substitution PF:

$$Y = f\left(K,L\right) = A\left(bK^{-\rho} + \left(1-b\right)L^{-\rho}\right)^{-\delta/\rho}, \rho = \frac{1-\sigma}{\sigma}, 0 \le \sigma \le 1, \sigma = const. \quad (1)$$

- Cobb - Douglas PF:

$$Y = f\left(K,L\right) = A\left(K\right)^{\alpha}\left(L\right)^{\beta}, \alpha > 0, \beta > 0, \alpha + \beta = 1, \sigma = 1. \quad (2)$$

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- Variable elasticity substitution PF, the construction procedure of which is described in (Sokol & Kutyshkin, 2016; Sokol et al., 2017).

In expressions (1.2), K, L are the values of the factors of capital and labor costs in the economic subsystem of the RSES under consideration; α, β - elasticity of the final product Y by cost factors K, L; δ is an indicator of the uniformity of the FS: δ = 1; σ is the elasticity of substitution of the cost factor K by cost factor L; A - a constant, the values of which, like the values of the variables α, β, σ, are determined by the methods of mathematical statistics.

To describe the cost factors in the construction of PF, as a rule, use the following economic indicators (Sokol et al., 2017). For the capital cost, factor K is the cost of fixed assets of the RSES economic subsystem without or taking into account their load, as well as investments in its fixed capital. For the factor of labor costs L, the average number of people employed in the economy of the region, labor costs of workers employed in the region's economy, actual hours worked in the sectors of the region's economy are used. To solve problems of an analytical nature with an appropriate justification, the construction of a production function allows the use of combinations of all of the above economic indicators. If it is necessary to solve the tasks of forecasting GRP, then it is possible to use only those indicators for which models for assessing changes in their values are developed and tested taking into account the adopted forecasting horizons. In this context, from the above indicators for L, it is advisable to use only the indicator of the average number of people employed in the region's economy, and for K, the cost of its fixed assets.

Changes in the values of the factor of labor costs L are proposed to evaluate the dependence (Kolemaev, 2005):

$$L_{t.calc} = L_{0.act}\, exp(\nu t),\ t \in [0,\ldots,T]. \quad (3)$$

Here Lt.calc, L0.act - the calculated and actual values of the average number of people employed in the regional economic subsystem of the RSES at the times t and t = 0 (for the initial year of the considered retrospective time period (t = [0, ..., T] of the functioning of the RSES) respectively; ν is a constant determined by the methods of mathematical statistics in the analysis of retrospective data L0.act for the same period of time.

Changes in the values of the capital cost factor K for the period t = [0,…,T], in turn, are proposed to be determined by the dependence (Kolemaev, 2005):

$$K_{t.calc} = (1 - \mu_{t-1.act})\, K_{t-1.act} + In_{t-1.act}, \quad (4)$$

where Kt.calc, Kt-1.act - the estimated and actual cost of fixed assets of the economic subsystem of the RSES in the year t and t-1, respectively; Int-1.act - the actual value of investments in fixed assets of the RSES economic subsystem in year t-1; μt-1.act - the actual coefficient characterizing the disposal of fixed assets of this subsystem at the end of the year t-1.

PF identification is carried out by presenting Y, K and L in the form of a series of basic indices relative to the selected year T0, the retrospective series of the source data of the corresponding economic indicators under consideration. If for the GRP value Y and the number of employees in the regional economic subsystem the formation of the basic indices IY, IL is not particularly difficult, then it is

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advisable to use the methodology proposed in (Bessonov & Voskoboinikov, 2006) to determine the values of the basic indices of the physical volume of the value of the basic production assets of the economic system I_K.

4. Purpose of the Study

This paper considers a number of issues related to the use of production functions for short-term forecasting of the gross regional product of a regional socio-economic system using the example of the RSES of the Khanty-Mansiysk Autonomous Okrug - Yugra (RSES KhMAO - Yugra).

5. Research Methods

Short-term forecasting of the GRP is proposed to be carried out as a result of the following sequence of procedures:

1. For each $t \in [0,\ldots,T]$ according to the official socio-economic statistics presented in a regional context, the values of economic indicators of the final product Yt.ф f (GRP), cost factors of production Kt.act, Lt.act and the value of investments in fixed assets Int.act are identified for considered economic subsystem RSES.

2. The sequences of absolute values of Yt.act, Kt.act and Lt.act are converted into series of basic indices IY.act(t), IK act(t) and IL.act (t).

3. For the time interval t = [0,…,T] constructing the CES PF (1) includes the CD PF (2) (Sokol & Kutyshkin, 2016), and the VES PF (Sokol et al., 2017).

4. According to (3.4), for t = T+1, the calculated values KT+1.calc and LT+1.calc are determined, which are being transformed by the basic indices IK.calc(T+1) and IL.calc(t) of the corresponding series formed earlier indices (paragraph 2).

5. Using the constructed PFs (paragraph 3), the GRP index IY.calc(T+1) is calculated based on the values of IK.calc(T+1) and IL.calc(t).

6. The relative forecast error of the gross regional product of the RSES economic subsystem under consideration is determined:

$$\varepsilon(T+1) = \frac{I_{Y.calc}(T+1) - I_{Y.act}(T+1)}{I_{Y.act}(T+1)}. \quad (5)$$

The initial statistics of GRP values (Yt.act), cost factors of production Kt.act, Lt.act, as well as the values of investments in fixed assets Int.act are given in the statistical compilation “Regions of Russia Social-economic indicators ”(https://www.gks.ru/folder/210/document/13204). The absolute values of Yt.act, Kt.act, Lt.act according to paragraph 2 of the previous section were converted into series of basic indices of the physical volume IY.act(t), IK.act(t), IL.act(t). The year 2001 was chosen as the base one (t = 0). Table 1 shows the values of these indices, which were subsequently used to identify PF.

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Table 1. The basic indices $I_{Y.\,act}(t)$, $I_{K.\,act}(t)$, $I_{L.\,act}(t)$ of the economy of the RSES KhMAO - Yugra from 2001 to 2017

Year	$I_{Y.act}(t)$	$I_{K.act}(t)$	$I_{L.act}(t)$	Year	$I_{Y.act}(t)$	$I_{K.act}(t)$	$I_{L.act}(t)$
2001	1,0000	1,0000	1,0000	2010	1,5082	1,8655	1,0333
2002	1,0250	1,0996	1,0107	2011	1,5218	1,9857	1,0402
2003	1,1193	1,1813	1,0128	2012	1,4929	2,1272	1,0501
2004	1,2077	1,2598	1,0055	2013	1,4944	2,2782	1,0546
2005	1,3575	1,3421	1,0097	2014	1,4944	2,4312	1,0558
2006	1,4484	1,4233	1,0123	2015	1,4788	2,5764	1,1942
2007	1,4948	1,5265	1,0137	2016	1,4788	2,7101	1,1804
2008	1,5381	1,6364	1,0250	2017	1,5098	2,8463	1,2408
2009	1,4643	1,7535	1,0330				

The data in Table 1 regarding the baseline index of the average number of people employed in the economy $I_{L.act}(t)$ show the presence of significant "jumps" of this indicator in the interval of 2012 -2017 against the background of stagnation in the value of $I_{Y.act}(t)$ and a uniform increase in $I_{K.act}(t)$. Table 2 shows the results of constructing models (3) of changes in the average number of people employed in the RSES KhMAO - Yugra economy at $L_{0.act}$ = 868,7 thousand people.

Table 2. Parameters of the model for changing the average number of employees (3) in the regional economic system of the RSES KhMAO - Yugra from 2001 to 2016

Time interval, year	Values ν of the equation (3)	R^2	F_{calc}	The significance of the criterion F
2001 – 2012	0,0037	0,9555	215,04	$1{,}373 \cdot 10^{-7}$
2001 – 2013	0,0039	0,9626	283,51	$1{,}146 \cdot 10^{-8}$
2001 – 2014	0,004	0,8868	390,38	$6{,}089 \cdot 10^{-10}$
2001 – 2015	0,0056	0,7217	33,716	$8{,}386 \cdot 10^{-5}$
2001 – 2016	0,0066	0,7528	42,637	$1{,}912 \cdot 10^{-5}$

The coefficients of disposal of fixed assets $\mu_{t.act}$ RSES KhMAO - Yugra according to the statistical collection "Regions of Russia Socio-economic indicators" are equal: $\mu_{2012.act}$ = 0,6; $\mu_{2013.act}$ = 0,7; $\mu_{2014.act}$ = 0,7; $\mu_{2015.act}$ = 0,8; $\mu_{2016.act}$ = 0,8.

6. Findings

Identification of the above PFs and prediction of $I_{Y.calc}(T+1)$ (item 2) values were carried out in MatLab15 ™. Table 3 presents the identified Cobb-Douglas production functions (2) for each of the considered time intervals. Here are the values of the determination coefficient R^2 for the identified PFs and the corresponding calculated values of the Fisher criterion F_{calc}, as well as tabular values of this criterion F_T for the significance level of 0.05 and two degrees of freedom (Korolyuk et al., 1985).

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Table 3. Identified Cobb-Douglas-type PF variables for economics of the RSES KhMAO - Yugra from 2001 to 2016

Time interval, years	Cobb-Douglas PF parameters	R^2	F_{calc}/F_T
2001 – 2012	A = 1,0541; α = 0,5802; β = 0,4198.	0,7579	31,31/3,88
2001 – 2013	A = 1,0748; α = 0,5023; β = 0,4977.	0,6992	25,57/3,80
2001 – 2014	A = 1,0923; α = 0,4460; β = 0,5540.	0,6808	25,59/3,74
2001 – 2015	A = 1,1133; α = 0,3755; β = 0,6245.	0,5627	16,72/3,68
2001 – 2016	A = 1,1302; α = 0,3240; β = 0,6760.	0,4880	13,37/3,63

Table 4, the structure of which is similar to Table 3, in turn, presents the results of constructing CES production functions (1) for the same time intervals (values of the F_T criterion for the significance level of 0.05 and three degrees of freedom (Korolyuk et al., 1985).

Table 4. Identified values of variables of the CES PF for economics of the RSES KhMAO - Yugra from 2001 to 2016

Time interval, years	CES PF parameters	R^2	F_{calc}/F_T
2001 – 2012	A = 1,0343; b = 0,89; σ = 0,1436.	0,8657	64,46/3,49
2001 – 2013	A = 1,0381; b = 0,9; σ = 0,1364.	0,8560	65,36/3,41
2001 – 2014	A = 1,0407; b = 0,91; σ = 0,1320.	0,8497	67,81/3,34
2001 – 2015	A = 1,0450; b = 0,9; σ = 0,1218.	0,7468	46,33/3,29
2001 – 2016	A = 1,0466; b = 0,88; σ = 0,1207.	0,7855	51,27/3,24

Identification of PFs of type VES - a function according to the methodology [4] allows you to create only a tabular form for representing this function for the considered time interval. Due to the limitations of the scope of this work, the authors cite only one such table for the interval of 2001–2016 (table 5). Relative error ε_t between $I_{Y.act}$ and $I_{Y.calc}$ (PF VES) calculated using the constructed PF VES is determined by the dependence, the structure of which is similar to (5).

Table 5. The results of approximating the values of the GRP of the RSES KhMAO - Yugra from PF type VES – function (Sokol & Kutyshkin, 2016)

Year	$I_{Y.act}$	$I_{Y.calc}$(PF VES)	ε_t	σ
2001	1,0000	0,9990	0,0011	0,0007
2002	1,0250	1,0983	0,0715	0,00071
2003	1,1193	1,1732	0,0482	0,0075
2004	1,2077	1,2385	0,0255	0,1483
2005	1,3575	1,3047	0,0389	0,1739
2006	1,4484	1,3661	0,0568	0,1971
2007	1,4948	1,4249	0,0468	0,0847
2008	1,5381	1,4408	0,0633	0,02951

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2009	1,4643	1,4522	0,0082	0,02951
2010	1,5082	1,4527	0,0368	0,0041
2011	1,5218	1,4625	0,0390	0,00054
2012	1,4929	1,4766	0,0109	0,00004
2013	1,4944	1,4830	0,0076	0,00004
2014	1,4944	1,4848	0,0064	0,00004
2015	1,4788	1,6795	0,1357	0,00004
2016	1,4788	1,6597	0,1223	0,00004

Relative error $\overline{\varepsilon_t}$ = 0,0449. Coefficient R^2 = 0,7871.

The calculated values of the elasticity of substitution of production cost factors σ for the considered RSES show that starting from 2011 the used VES PF is very close to the Leontyev PF, for which σ → 0. Table 6 together shows:

- statistics on GRP $I_{Y.act}(T+1)$ for the corresponding year for which GRP forecasting is carried out (table 1);
- results of forecasting the GRP value of $I_{Y.calc}(T+1)$ using the PF Cobb-Douglas, CES PF (tables 3,4) and PF VES (including table 5).

Table 6. The predicted values of GRP of RSES KhMAO – Yugra in the period from 2013 to 2017, obtained using the PF type CES and VES functions

Year	$I_{Y.act}(T+1)$	$I_{Y.calc}(T+1)$, PF CD	$\varepsilon(T+1)$, PF CD	$I_{Y.calc}(T+1)$, PF CES	$\varepsilon(T+1)$, PF CES	$I_{Y.calc}(T+1)$, PF VES	$\varepsilon(T+1)$, PF VES
2013	1,4943	1.7319	0,1592	1,5696	0,0504	1,5272	0,0220
2014	1,4944	1,7213	0,1519	1,5658	0,0478	1,5286	0,0229
2015	1,4788	1,6687	0,1467	1,5563	0,0524	1,5271	0,0326
2016	1,4788	1,7112	0,1571	1,5693	0,612	1,5565	0,0525
2017	1,5098	1,7089	0,1556	1,6150	0,0912	1,5860	0,0724

The calculated values of $\varepsilon(T+1)$ for all used PF show steady growth. This is due to the fact that since 2012 year, in the economic system under consideration, against the background of the growth in the capital-labor ratio $k_{t.act} = I_{K.act} / I_{L.act}$, the growth in the average labor productivity $y_{t.act} = I_{Y.act} / I_{L.act}$ has practically stopped. This is also evidenced by the values of $I_{Y.act}$ and $I_{L.act}$ Table 1. The construction of neoclassical linearly homogeneous PFs provides for the fulfillment of the requirement associated with their growth (the first derivative of Y with respect to cost factors K and L must be greater than zero), as well as ensuring the convexity of this function upward (the second derivative of Y with respect to K and L should be less than zero). As a result of this, the construction of PFs based on $I_{Y.act}$ data, which are characterized by a fairly stable downward trend (2011 – 2016 years), leads to an increase in the discrepancy between the calculated $I_{Y.act}$ and source $I_{Y.act}$ data.

7. Conclusion

The results obtained (table 6) show that forecasting the GRP values of the considered RSES using a VES type PF - the function gives a better approximation of the calculated values of this economic

indicator to its actual values, which are presented in the materials of the Federal State Statistics Service of the Russian Federation (Rosstat). It should also be noted that the use of the apparatus of production functions to describe economic systems in the functioning of which there is stagnation of the values of the final product with a steady increase in the costs of production factors will not give a close approximation of the calculated values of the final product to its actual values. At the same time, the authors believe that short-term forecasts of the GRP value using PFs of the VES-type function can be considered an upper estimate of the values of this indicator.

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