

AMURCON 2020
International Scientific Conference**IMPROVEMENT OF APPROXIMATION DEPENDENCES OF
DEFORMATION DIAGRAMS OF CONCRETE OF DIFFERENT
STRENGTH**

V. L. Zemlyak (a), A. S. Vasilyev (b)*, O. A. Odinkova (c), O. Ya. Dubey (d)
*Corresponding author

- (a) Sholom-Aleichem Priamursky State University, 70a Shirokaya St., Birobidzhan, Russia, vellkom@list.ru
(b) Sholom-Aleichem Priamursky State University, 70a Shirokaya St., Birobidzhan, Russia, Vasil-grunt@mail.ru
(c) Pacific National University, 136, Tihokeanskaya St., Khabarovsk, Russia, odi37@mail.ru
(d) Sholom-Aleichem Priamursky State University, 70a Shirokaya St., Birobidzhan, Russia, olesya_dubey@mail.ru

Abstract

The analysis technique of statically undeterminable framed/ lattice structures, both physically and geometrically nonlinear systems, factoring in actual reinforced concrete stiffness matrices, allows taking into account several factors. They influence the deformability/stress-strain capacity of the system, which are not considered when applied for calculating by elastic methods. When computing the strength and deformability of the efforts in cross-section, which is normal to the longitudinal axis of the element, it is determined according to a nonlinear deformation model using the balance equations of external forces and internal forces in the section of the element. Besides, the connection between the axial strain and relative deformations of concrete and reinforcement is accepted as diagrams of the status (deformation) of concrete and reinforcement. For this purpose, the researchers pre-test materials for single-axis and biaxial tension, compression, and bending. The paper compares the diagrams obtained as a result of applying this dependence to approximated and experimental diagrams of other authors. The comparison is performed for concretes with different mechanical characteristics, as well as for epoxy-polyamide films. In such a case, the elastic part of the compression diagram is characterized by a straight/right line. The segment where the material demonstrates creeping and non-linearity properties can be interpolated using the polynomial of degree 3 proposed by the author. Finally, it was found out that the piecewise linear dependence that the authors proposed allows one to obtain diagrams with satisfactory accuracy that agree with the other authors' experimental data.

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Keywords: Deformation diagrams of materials, matrix of mechanical characteristics, approximation, system of equations



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1. Introduction

The analysis technique of statically undeterminable framed/ lattice structures, both physically and geometrically nonlinear systems, factoring in actual reinforced concrete stiffness matrices, allows taking into account some factors. They influence the deformability/stress-strain capacity of the system, which are not considered when applied for calculating by elastic methods. When computing the strength and deformability of the efforts in cross-section, which is normal to the longitudinal axis of the element, it is determined according to a nonlinear deformation model using the balance equations of external forces and internal forces in the section of the element.

Moreover, the connection between the axial strain and relative deformations of concrete and reinforcement is accepted as diagrams of the status (deformation) of concrete and reinforcement. The article presents the author-developed approximation dependences for the deformation diagrams of concrete in different strength range. Then, using these dependences, one can integrate them in algorithms for numerical calculations of structures made of brittle, structurally heterogeneous materials similar to concrete (Kozin et al., 2019; Zemlyak et al., 2019). A lot of studies review concrete break and cracking as well as drawing deformation diagrams (Balan et al., 2001; Belyaev et al., 2017; Filatov & Suvorov, 2016; Karpenko et al., 2015; Kochkarev & Galinska, 2017; Prado & Mvan Mier, 2003).

2. Problem Statement

The main objectives were the following during the research:

- To provide a literature review of experimental studies and approximation dependencies of concrete deformation diagrams.
- To suggest approximation dependencies based on piecewise linear interpolation.
- To test the activity of the dependencies offered by comparing them to deformation diagrams in tension and compression for materials with different mechanical characteristics

3. Research Questions

3.1. Material non-linearity accounting within a heterogeneous environment

As is known, physical nonlinearity is caused by a disproportionate connection between deformations and strains (Klovanich & Bezushko, 2009):

$$\{\sigma\{\varepsilon\}\} = [D\{\varepsilon\}]\{\varepsilon\}, \quad \{\sigma\{\varepsilon\}\}^{i+1} = [D\{\varepsilon\}]^i \{\varepsilon\}^{i+1} \quad (1)$$

where $[D\{\varepsilon\}]_i$ is a matrix of mechanical characteristics of the material at the I-th load step. The dependence between deformations and strains can be taken into account using diagrams of material deformation in case of short-term strength (Klovanich & Bezushko, 2009). In this paper, the authors examine the dependencies that combine relative deformations with strains ($\varepsilon_b - \sigma_b$) at monoaxial tension and compression. It is worth noting that reinforced concrete and glass-fibre plastic can be illustrating materials that characterize the operation of heterogeneous environment. The strength and deformability of

reinforcing steels are attributed to the $\sigma - \varepsilon$ diagram in figure 1-b. In this paper, (Taranukha & Vasilyev, 2015) the author offers to use piecewise linear functions. Meanwhile,

$$\varepsilon_{s,el} = \frac{R_s}{E_s}, \quad E_{s2} = \frac{\sigma_{s,y} - R_s}{\varepsilon_{s,int} - \varepsilon_{s,y}} \quad (2)$$

Here $\sigma_{s,y}$ is temporary armature resistance, R_s is its design strength, E_s is a modulus of armature elasticity.

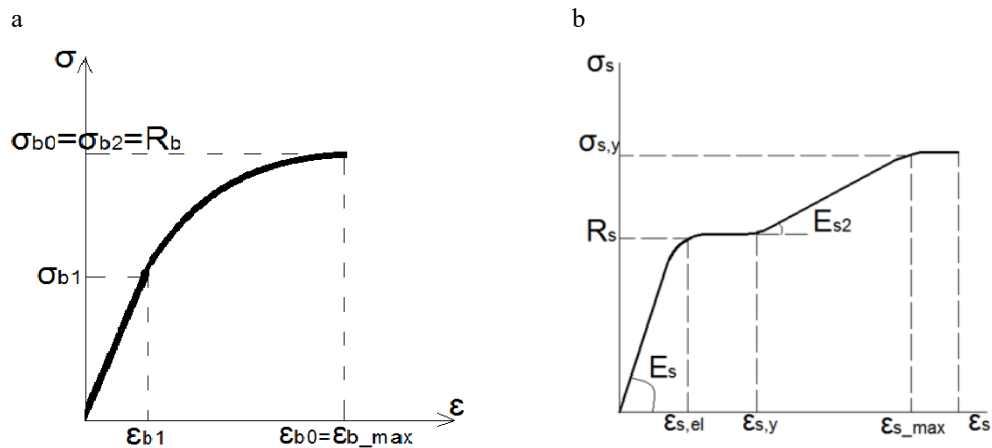


Figure 1. Diagrams of material deformation in a heterogeneous environment with a metallic through the example of reinforced concrete: a-binder (concrete), b-reinforcing elements (reinforcement)

4. Purpose of the Study

The research aims at developing approximation dependences of deformation diagrams that describe most minutely the brittle material deformation, such as concrete/

5. Research Methods

Approximation dependences for diagrams of material deformation:

Figure 1- a shows a uniaxial compression deformation diagram for a binder in a heterogeneous environment. A similar curve is described by a large number of different materials. The elastic part of the deformation diagram at $0 \leq \varepsilon_b \leq \varepsilon_{b1}$ is characterized by a straight line with coordinates $(0;0)$, $(0.55R_b/E_0; 0.55R_b)$. The part of the function $\varepsilon_{b1} \leq \varepsilon_b \leq \varepsilon_{b0}$ where the material demonstrates creeping and non-linearity properties can be interpolated using a polynomial.

$$\sigma_b(\varepsilon_b) = a_1 \varepsilon_b + b_1 \varepsilon_b^2 + c_1 \varepsilon_b^3 + d_1 \varepsilon_b^4, \quad E(\varepsilon_b) = \frac{\sigma_b(\varepsilon_b)}{\varepsilon_b} = a_1 + b_1 \varepsilon_b + c_1 \varepsilon_b^2 + d_1 \varepsilon_b^3 \quad (3)$$

It is necessary to solve a system of the following equations to determine the coefficients of a polynomial:

$$0.55R_b = a_1 * \frac{0.55R_b}{E_0} + b_1 \left(\frac{0.55R_b}{E_0} \right)^2 + c_1 \left(\frac{0.55R_b}{E_0} \right)^3 + d_1 \left(\frac{0.55R_b}{E_0} \right)^4,$$

$$\frac{E_0 \varepsilon_{b_max} + Rb}{2\varepsilon_{b_max}} = a_1 + 2b_1 \left(\frac{0.55Rb}{E_0} \right) + 3c_1 \left(\frac{0.55Rb}{E_0} \right)^2 + 4d_1 \left(\frac{0.55Rb}{E_0} \right)^3, \quad (4)$$

$$Rb = a_1 \varepsilon_{b_max} + b_1 \varepsilon_{b_max}^2 + c_1 \varepsilon_{b_max}^3 + d_1 \varepsilon_{b_max}^4,$$

$$0 = a_1 + 2b_1 \varepsilon_{b_max} + 3c_1 \varepsilon_{b_max}^2 + 4d_1 \varepsilon_{b_max}^3,$$

where a1, b1, c1, d1 are the coefficients of the polynomial, Rb is the material strength at a uniaxial compression, E0 is the initial compression modulus, ε_{b_max} is the maximum deformations in the material deformation diagram, $\varepsilon_{b_max} = \varepsilon_0$.

Likewise, for a stress-strain diagram, the elastic part at the interval $0 \leq \varepsilon_{bt} \leq \varepsilon_{bt1}$ is characterized by a straight line with coordinates (0;0), (0.55Rbt/E0; 0.55Rbt). The segment $\varepsilon_{bt1} \leq \varepsilon_{bt} \leq \varepsilon_{bt0}$, where the material demonstrates creeping and non-linearity properties, is interpolated by the polynomial.

$$\sigma_{bt}(\varepsilon_{bt}) = a_2 \varepsilon_{bt} + b_2 \varepsilon_{bt}^2 + c_2 \varepsilon_{bt}^3 + d_2 \varepsilon_{bt}^4, \quad E(\varepsilon_{bt}) = \frac{\sigma_{bt}(\varepsilon_{bt})}{\varepsilon_{bt}} = a_2 + b_2 \varepsilon_{bt} + c_2 \varepsilon_{bt}^2 + d_2 \varepsilon_{bt}^3 \quad (5)$$

For this reason, one needs to solve a system composed of the following equations:

$$0.55Rbt = a_2 * \frac{0.55Rbt}{E_0} + b_2 \left(\frac{0.55Rbt}{E_0} \right)^2 + c_2 \left(\frac{0.55Rbt}{E_0} \right)^3 + d_2 \left(\frac{0.55Rbt}{E_0} \right)^4,$$

$$\frac{E_0 \varepsilon_{bt_max} + Rbt}{2\varepsilon_{bt_max}} = a_2 + 2b_2 \left(\frac{0.55Rbt}{E_0} \right) + 3c_2 \left(\frac{0.55Rbt}{E_0} \right)^2 + 4d_2 \left(\frac{0.55Rbt}{E_0} \right)^3, \quad (6)$$

$$Rbt = a_2 \varepsilon_{bt_max} + b_2 \varepsilon_{bt_max}^2 + c_2 \varepsilon_{bt_max}^3 + d_2 \varepsilon_{bt_max}^4,$$

$$0 = a_2 + 2b_2 \varepsilon_{bt_max} + 3c_2 \varepsilon_{bt_max}^2 + 4d_2 \varepsilon_{bt_max}^3,$$

where a2, b2, c2, d2 are the coefficients of the polynomial that are deduced from the solution of the system of equations (6), similar to the coefficients in compression. At the interval $\varepsilon_{bt0} \leq \varepsilon_{bt} \leq \varepsilon_{bt2}$ the relation $\varepsilon_{bt} - \sigma_{bt}$, is approximated by a straight line with the coordinates (ε_{bt_max} , Rbt), (0.00015, Rbt).

6. Findings

It is worthwhile to say that the descending/ downward part of the chart is not taken into account. Figure 1-a shows a deformation diagram that describes the deformation of many different materials. Stress-strain diagrams are drawn in a similar way to compression diagrams. Also, the calculated values of the binder (concrete) uniaxial compression resistance Rb are replaced with the calculated values of the binder (concrete) uniaxial tension resistance Rbt.

Table 1 shows materials with different mechanical characteristics corresponding to different concrete classes.

Table 1. Mechanical characteristics

Specimen number	Elastic modulus, Pa	Uniaxial Compression Strength, Pa	Uniaxial tensile strength, Pa
Specimen 1	3×10^{10}	20×10^6	1.75×10^6
Specimen 2	3.7×10^{10}	32×10^6	2.25×10^6
Specimen 3	3.95×10^{10}	43×10^6	2.75×10^6

Figure 2 shows the deformation diagrams based on the data of approximation dependences compared with the diagrams of Karpenko (1996), as well as simplified diagrams from a series of regulations for calculations of concrete and reinforced concrete structures.

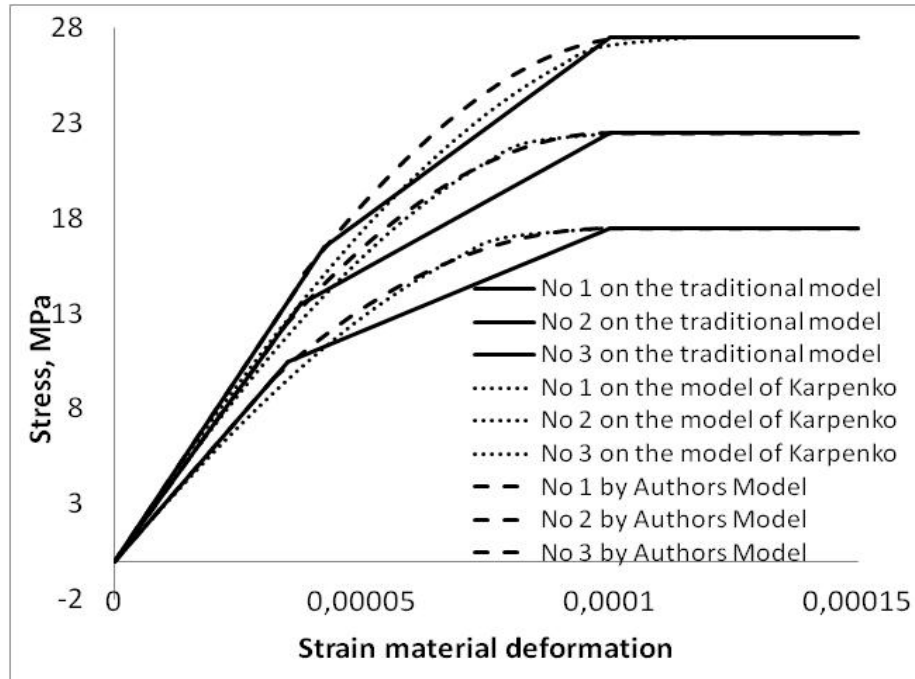


Figure 2. Comparison of compression deformation diagrams for each sample according to SP63. 13330. 2012 model, the model of N. I. Karpenko, and the author's model.

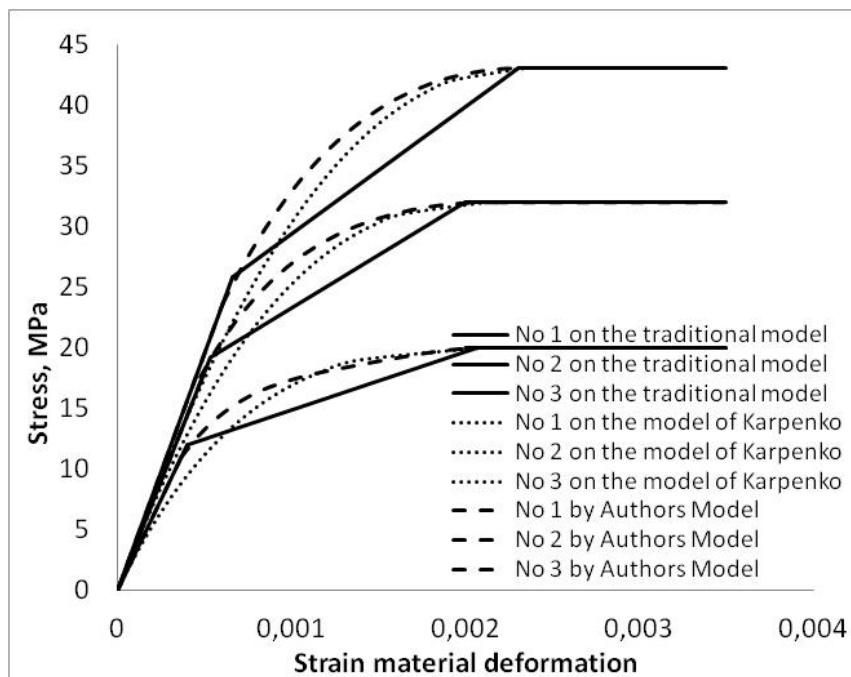


Figure 3. Comparison of tensile strain diagrams for each sample according to BR 63. 13330.2012 model, the model of N. I. Karpenko, and the author's model.

Proposing N. I. Karpenko's diagrams (Figure 3) as the benchmark, the authors emphasize that there is a good agreement of diagrams in tension and compression for concretes with different mechanical characteristics.

7. Conclusion

The obtained approximation dependences correspond satisfactorily to the other authors' diagrams. These dependencies can be used to calculate the stress-strain state of reinforced concrete structures at various loading stages.

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