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FINITE ELEMENT FOR STRUCTURAL ANALYSIS OF FRAGILE STRUCTURALLY HETEROGENEOUS REINFORCED MATERIALS

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Abstract

The constructions made of fragile, structurally heterogeneous materials should be estimated according to the limit states criteria both using methods adopted from standard-setting instruments, and numerical algorithms from the laws of construction mechanics, in particular applying the finite element method (FEM) by the stepping and iterative calculations and the main principles of solid mechanics. In this paper, the researchers propose a new finite element for calculating structures made of a heterogeneous medium neighbouring with a metal component. The finite element allows one to take into account the amounts of materials in its stiffness matrix. This finite element also contains information about the physical nonlinearity of its constituent materials. Nonlinearity is considered based on material strain diagrams. The new stiffness matrix for a heterogeneous nonlinear finite element is inspired by the idea of combining the amounts of materials by that of the entire finite element. Broadly described, the bottom line is that the characteristics of materials, such as modulus of elasticity and Poisson’s ratio, are replaced by specific, heterogeneous characteristics. Computer programs were designed and registered to calculate structures from heterogeneous media. The authors calculated numerically a structure made of heterogeneous material with a metal component and attained distinctive results by this investigation. They also could conclude about the ways the proposed stiffness matrix might be applied.

Keywords: Heterogeneous medium, metal component, finite element, stiffness matrix, mathematical model

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1. Introduction

As is widely known, the heterogeneous medium provides a basis for composite materials that are being currently actively used in various modern branches of industry: construction, automotive and shipbuilding. It is worth calculating structures made of such materials according to the criteria of limit states, both using methods from normative documents, and numerical methods of structural mechanics. In particular, the finite element method using adopted from standard-setting instruments and numerical algorithms from the laws of construction mechanics. In particular, the finite element method (FEM) is applied by the stepping and iterative calculations and the main principles of solid mechanics. Calculations of composite structures are solved numerically with a computer and observed in the researches of different authors (Klovanich & Bezushko, 2009; Kozin et al., 2019; Taranukha & Vasilyev, 2015; Zemlyak et al., 2019).

There are two approaches to composite modelling: macro- and micro-mechanical. The macro-mechanical approach involves an analysis of heterogeneous materials as those with anisotropic properties, having different mechanical performance on the various dimensions. The model is designed in global coordinate axes, and the location of particular materials within a heterogeneous medium is not noticed. In this case, the permitted strains for the composite material are determined through extensive testing. This makes computing easier. However, usually, when using the macro-mechanical approach, the strains in the material do not exceed the safety margin, and the data needed for calculations are taken from experiments. However, as the book by Aboudi et al. (2012) suggests, the use of the macro-mechanical approach might have difficulty when doing nonlinear calculations and trying to predict destruction and structural damage. First of all, this is caused by the fact that the destruction of anisotropic material in different directions requires extensive testing. Besides, a large number of experiments to obtain appropriate mechanical characteristics in various ways of anisotropy. In the paper (Feyel & Chaboche, 2000), the authors discuss modelling the behaviour of structures with a long-fibre composite material SiC / Ti reinforcement with a regular microstructure. A new multi-scale model of the behaviour of composites based on the multi-level finite element method (FEM) is used to account for inhomogeneities in the function between the fibre and the matrix.

Structural and phenological approaches are also widely used for calculating reinforced concrete as a composite material (Balan et al., 2001; Dovzhyk et al., 2020; Karpenko, 1996; Piloto et al., 2020; Stefan et al., 2019; Vatulia et al., 2019).

On the contrary, the micro-mechanical approach takes into account the geometry and location of materials in the composite. The main idea is to predict the behaviour of heterogeneous material according to the effect of its components and their placement when loading on structural elements. Such an approach allows one not only to explore existing materials but also to design new composite materials. In the case of non-linearity and destruction, micromechanics of composites investigates individual materials in their local area, where they are destructed.

2. Problem Statement

The following study objectives have been pursued during the survey:

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- To provide a literature review of numerical studies of structures made of structurally heterogeneous materials.
- To develop a stiffness matrix for a physically nonlinear composite finite element.
- To compare calculations according to the proposed final element to calculations from other sources.

3. Research Questions

In this paper, the authors suggest a new finite element for calculating structures from a heterogeneous medium with a metal component. The finite element allows one to consider a fairly large number of materials in its stiffness matrix. This finite element also contains information about the physical non-linearity of its constituent materials. Non-linearity is taken into account based on material load-strain diagrams. The new stiffness matrix for a heterogeneous nonlinear finite element is based on the idea of combining the amounts of materials by one of the entire finite element. The core of this idea is that the characteristics of materials, such as modulus of elasticity and Poisson's ratio, are replaced by special, heterogeneous properties. Using this finite element, one can compute structures that combine the multi-user processing of various materials, including composite ones. Calculations can be done in a non-linear model, when the load on the structure changes, before a limit state happens in the structure or its destruction. The criterion for the limit state happening in the finite element is an exceedance of general stresses arising in the finite element over the permissible strains.

4. Purpose of the Study

The purpose of the study is to propose a stiffness matrix of a new composite finite element and carry out a calculation of a building structure made of a fragile structurally heterogeneous composite material based on it

5. Research Methods

5.1. The stiffness matrix of the special non-linear finite element for the heterogeneous medium

Several materials cooperate by a transfer to a composite finite element in the form of a parallelepiped and calculating its stiffness matrix. In this case, modulus of elasticity and Poisson's ratio are considered as some given characteristics that depend simultaneously on the corresponding characteristics of both (all) materials that make up the composite structure. The idea of combining amounts is used to define these characteristics:

$$E_{het} = \frac{E(\varepsilon_x)_1 V_1 + E(\varepsilon_x)_2 V_2 + ... + E(\varepsilon_x)_n V_n}{V_1 + V_2 + ... + V_n}, \quad \mu_{het} = \frac{(\mu_1)_x V_1 + (\mu_2)_x V_2 + ... + (\mu_n)_x V_n}{V_1 + V_2 + ... + V_n} \qquad (1)$$

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where E_1, E_2, ... E_n espectively, are the nonlinear secant modulus of elasticity of a material in the composite CE; V_1, V_2, ...V_n respectively, are the volumes of each material in the composite CE (where n can be rather large), $(\mu_1)_x$, $(\mu_2)_x$ и $(\mu_n)_x$ respectively, are Poisson's ratios for each material in the composite CE. Substituting (1) into the standard elasticity matrix, the elasticity matrix for a composite nonlinear bulk finite element *[D]$_{het}$* that combines co-working of different media is obtained.

$$D_{\text{het}} = \frac{E_{\text{het}}(1-\mu_{\text{het}})}{(1+\mu_{\text{het}})(1-2\mu_{\text{het}})} \begin{bmatrix} 1 & \frac{\mu_{\text{het}}}{1-\mu_{\text{het}}} & \frac{\mu_{\text{het}}}{1-\mu_{\text{het}}} & 0 & 0 & 0 \\ \frac{\mu_{\text{het}}}{1-\mu_{\text{het}}} & 1 & \frac{\mu_{\text{het}}}{1-\mu_{\text{het}}} & 0 & 0 & 0 \\ \frac{\mu_{\text{het}}}{1-\mu_{\text{het}}} & \frac{\mu_{\text{het}}}{1-\mu_{\text{het}}} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\mu_{\text{het}}}{2(1-\mu_{\text{het}})} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\mu_{\text{het}}}{2(1-\mu_{\text{het}})} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\mu_{\text{het}}}{2(1-\mu_{\text{het}})} \end{bmatrix} \quad (2)$$

It is $[D]_{het}$ matrix that the features of this work are related to.

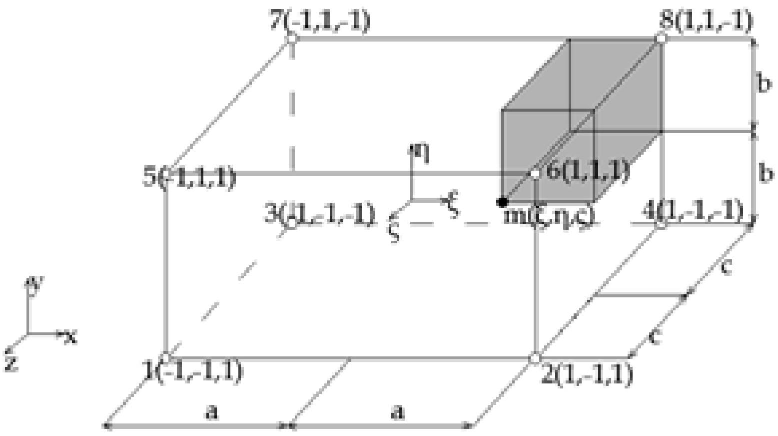

Figure 1. A finite element of a rectangular parallelepiped in normalized coordinates [4]

The finite element in the form of a rectangular parallelepiped contains eight nodes (Fig. 3). Deformation matrix [B] contains eight blocks. The standard block is equal to [4]:

$$B_i^{(k)} = [\Phi]C_k(\xi,\eta,\varsigma) = \frac{1}{8}\begin{bmatrix} \frac{\xi_k(1+\eta_k\eta)(1+\varsigma_k\varsigma)}{a} & 0 & 0 \\ 0 & \frac{\eta_k(1+\xi_k\xi)(1+\varsigma_k\varsigma)}{b} & 0 \\ 0 & 0 & \frac{\varsigma_k(1+\xi_k\xi)(1+\eta_k\eta)}{c} \\ \frac{\eta_k(1+\xi_k\xi)(1+\varsigma_k\varsigma)}{b} & \frac{\xi_k(1+\eta_k\eta)(1+\varsigma_k\varsigma)}{a} & 0 \\ 0 & \frac{\varsigma_k(1+\xi_k\xi)(1+\eta_k\eta)}{c} & \frac{\eta_k(1+\xi_k\xi)(1+\varsigma_k\varsigma)}{b} \\ \frac{\varsigma_k(1+\xi_k\xi)(1+\eta_k\eta)}{c} & 0 & \frac{\xi_k(1+\eta_k\eta)(1+\varsigma_k\varsigma)}{a} \end{bmatrix} \quad (3)$$

Furthermore, each block of the stiffness matrix is calculated using the formula:

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$$[K]_{ij}^{(k)} = abc\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}([B]^{(j)})^{T}[D]_{het}[B]^{(k)}\,d\xi d\eta d\varsigma \tag{4}$$

Here $[K]_i$ is the stiffness matrix of I-th CE; k and j are the numbers of CE nodes. Each element of the stiffness matrix $[K]^{k}_{i,j}$ consists of a third-order submatrix (Klovanich & Bezushko, 2009)

$$[K]_{i,j}^{(k)} = \begin{bmatrix} K_{1,1} & K_{1,2} & K_{1,3} \\ K_{2,1} & K_{2,2} & K_{2,3} \\ K_{3,1} & K_{3,2} & K_{3,3} \end{bmatrix} \tag{5}$$

Entering the variables k=1:8 и j=1:8, after drawing a certain integral (8), keeping in mind constancy of $[D]_{het}$ over the volume of the element, one gets:

$$\left.\begin{aligned} K_{1+n,1+m} &= \left[\begin{aligned} &\frac{1-\mu_{\text{hyb}}}{a^2}\xi_k\xi_j(1+\frac{1}{3}\eta_k\eta_j)(1+\frac{1}{3}\varsigma_k\varsigma_j)+\ldots \\ &\ldots+\frac{1-2\mu_{\text{hyb}}}{2}\left[\frac{1}{b^2}\eta_k\eta_j(1+\frac{1}{3}\xi_k\xi_j)(1+\frac{1}{3}\varsigma_k\varsigma_j)+\frac{1}{c^2}\varsigma_k\varsigma_j(1+\frac{1}{3}\xi_k\xi_j)(1+\frac{1}{3}\eta_k\eta_j)\right]\end{aligned}\right]\times\ldots \\ &\ldots\times\frac{a(E(\varepsilon_x)_1V_1+E(\varepsilon_x)_2V_2...+E(\varepsilon_x)_nV_n)}{32(1+\mu_{\text{hyb}})(1-2\mu_{\text{hyb}})} \\ K_{1+n,2+m} &= \frac{1}{ab}(1+\frac{1}{3}\varsigma_k\varsigma_j)\left[\frac{1}{2}\xi_k\eta_j+\mu_{\text{hyb}}(\xi_j\eta_k-\eta_j\xi_k)\right]\times\ldots \\ &\ldots\times\frac{a(E(\varepsilon_x)_1V_1+E(\varepsilon_x)_2V_2...+E(\varepsilon_x)_nV_n)}{32(1+\mu_{\text{hyb}})(1-2\mu_{\text{hyb}})} \end{aligned}\right\} \tag{6}$$

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The other elements of the stiffness matrix are defined in the same way. Here *n=1+3(j-1); m=1+3(k-1)*. As a result, one gets 24x24 CE stiffness matrix.

$$\sigma_{x,\text{арм}} = \frac{V_{\text{бет}}\dfrac{E(\varepsilon_x)_{\text{бет}}}{E_{0,\text{бет}}}+V_{\text{арм}}\dfrac{E(\varepsilon_x)_{\text{арм}}}{E_{0,\text{арм}}}}{V_{\text{арм}}\dfrac{E(\varepsilon_x)_{\text{арм}}}{E_{0,\text{арм}}}}\varepsilon_{x,het}E_{het} \tag{7}$$

In a general way, for any number of different media in a finite element, the strains in a particular material will be determined by the expression:

$$\sigma_{x,\text{k}} = \frac{\sum_{1}^{n}V_n\dfrac{E(\varepsilon_x)_{\text{n}}}{E_{0,\text{n}}}}{V_k\dfrac{E(\varepsilon_x)_{\text{k}}}{E_{0,\text{k}}}}\varepsilon_{x,het}E_{het} \tag{8}$$

The method of successive approximations was used as a method of the solution of nonlinear problems (Klovanich & Bezushko, 2009):

$$\left[\overline{K}(q^{n-1})\right]\left[q^n\right]=\left[\overline{P}\right] \tag{9}$$

where n is the number of the approximation stage.

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6. Findings

6.1. Numerical research of a statically determinable structure made of heterogeneous material with a metal component

Let's consider a reinforced concrete beam of a rectangular cross-section, studied in (Klovanich & Bezushko, 2009). The design scheme and discrete model of the beam are shown in figures 2A and 2b

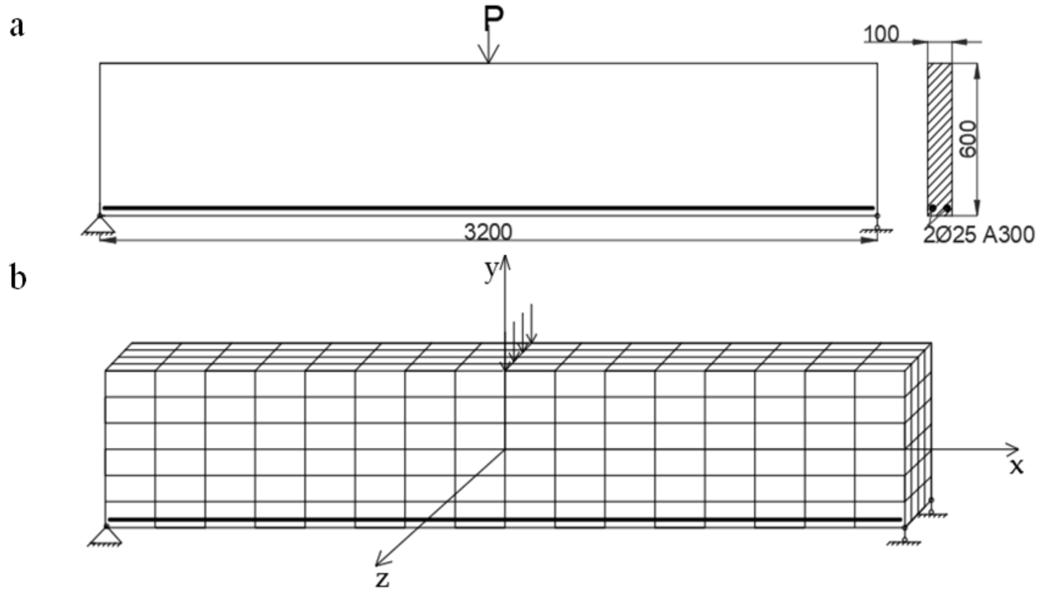

Figure 2. Beam model: a-design scheme and construction of a reinforced concrete beam, b-scheme of the finite element distribution

Since the beginning of level 1, the loading was performed by vertical forces of ΔP = 10 kN. Initial material characteristics are binder (concrete) Rb = 20 MPa, Rbt = 1.75 MPa, Eb = 30000 MPa, μ = 0.2, εb _max = 0.002, εbt_max = 0.0001; reinforcing element (rebar) Rs = 400 MPa, Es = 200000 MPa. The criterion for the strength of a reinforced concrete beam is the appearance of flowability in the reinforcing elements in the tension side of the beam.

Figure 6-a shows the change in the maximum movement in the beam at various loading stages, in comparison with the calculation of C. F. Klovanich by FEM (Klovanich & Bezushko, 2009). When the load-bearing capacity is lost, the normal strain values in the upper layers of the beam are -20 MPa. Figure 6-b illustrates the comparison of individual epures between the author's computation and the Concord PC.

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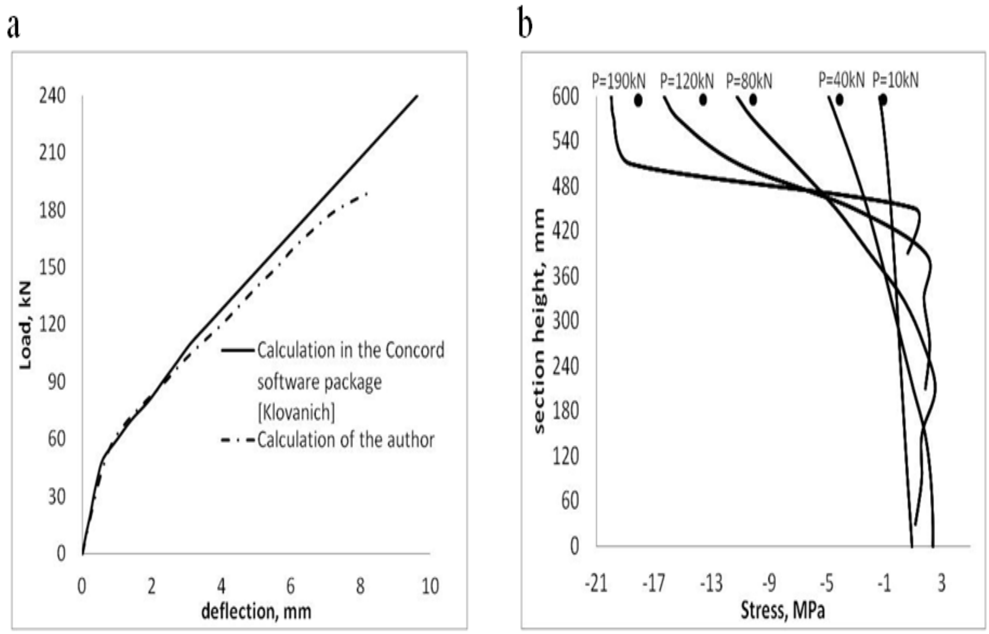

Figure 3. Comparison of computation results: a-comparison of load-deflection variations; b-comparison of some normal strain epures in the crosssection of the midspan of the beam (the author's computation with the upper-range values of similar epures obtained in the work of S. F. Klovanich in PC Concord [4])

Tables 1 and 2 compare the calculation results: load deflections at the destruction point and the structural critical load.

Table 1. Results of research

No	Maximum deflection			Error of results of Concord calculation [4]	
	Concord Calculation Δ, mm	Author's computation Δ, mm	Code specification Δ, mm	Author's computation %	Code specification %
1	9.62	8.34	9.3	13.31	3.32
2	Breaking load			Error of results of Concord calculation [4]	
3	Concord Calculation	Author's computation	Concord Calculation	Author's computation	Concord Calculation
4	P, kN	P, kN	P, kN	%	%
5	240	190	200	20.83	16.67

As one can notice, there can be observed a satisfactory concurrence of the obtained results.

7. Conclusion

The authors developed a mathematical model based on the approximation of material deformation diagrams and the stiffness matrix of a nonlinear composite finite element. This stiffness matrix takes into account changes in the mechanical characteristics of the materials included in it, depending on the tensely deformed condition at the previous stages of loading the structure.

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The researchers also generated the methods to define the stressed-strain state of composite structures at various loading stages, including the limit ones. This method allows the developers to calculate the collapse load and maximum movements of structures made of composite materials. However, one can take into account any number of materials in the composite.

References

Aboudi, J., Arnold, S. M., & Bednarcyk, B. A. (2012). *Micromechanics of Composite Materials*. Butterworth-Heinemann.

Balan, T. A., Spacone, E., & Kwon, M. (2001). 3D hypoplastic model for cyclic analysis of concrete structures. *Engineering Structures*, *23*, 333-342.

Dovzhyk, M., Bogdanov, V., & Nazarenko, V. (2020). Fracture of Composite Materials Under Compression Along Cracks. *Structural Integrity, 16*, 268-272. https://doi.org/10.1007/978-3-030-47883-4_48

Feyel, F., & Chaboche, J. -L. (2000). FE^2 multiscale approach for modelling the elastoviscoplastic behaviour of long fibre SiC/Ti composite materials. *Computer Methods in Applied Mechanics and Engineering, 183,* 309-330. https://doi.org/10.1016/S0045-7825(99)00224-8

Karpenko, N. I. (1996). *Obshchiye modeli mekhaniki zhelezobetona* [*General models of reinforced concrete* mechanics]. Stroyizdat. [in Russ.]

Klovanich, S. F., & Bezushko, D. I. (2009). *Metod konechnykh elementov v raschetakh prostranstvennykh zhelezobetonnykh konstruktsiy* [*Finite element method in the analysis of spatial reinforced concrete structures]*. ONMU. [in Russ.]

Kozin, V. M., Vasil'ev, A. S., Zemlyak, V. L., & Ipatov, K. I. (2019). Investigation of the limit state of ice cover under conditions of pure bending when using reinforcing elements. *Vestnik Tomskogo Gosudarstvennogo Universiteta. Matematika I Mekhanika, 61,* 61-69. https://doi.org/10.17223/19988621/61/6

Piloto, P. A. G., Balsa, C., Santos, L. M. C., & Kimura, E. F. A. (2020). Effect of the load level on the resistance of composite slabs with steel decking under fire conditions. *Journal of Fire Sciences, 38,* 212-231. https://doi.org/10.1177/0734904119892210

Stefan, R., Sura, J., Prochazka, J., Kohoutkova, A., & Wald, F. (2019). Numerical investigation of slender reinforced concrete and steel-concrete composite columns at normal and high temperatures using sectional analysis and moment-curvature approach. *Engineering Structures*, *190*, 285-305. https://doi.org/10.1016/j.engstruct.2019.03.071

Taranukha, N. A., & Vasilyev, A. (2015). Numerical investigation problems limit carrying capacity of composite structures, *Marine intellectual technologies*, *2*(3), 27-32.

Vatulia, G. L., Lobiak, O. V., Deryzemlia, S. V., Verevicheva, M. A., & Orel, Y. F. (2019). Rationalization of cross-sections of the composite reinforced concrete span structure of bridges with a monolithic reinforced concrete roadway slab. *IOP Conference Series: Materials Science and Engineering, 664*, 012014. https://doi.org/10.1088/1757-899X/664/1/012014

Zemlyak, V. L., Kozin, V. M., Vasil'ev, A. S., & Ipatov, K. I. (2019). Experimental and Numerical Investigations of the Influence of Reinforcement on the Load-Carrying Capacity of Ice Crossings. *Soil Mechanics and Foundation Engineering*, *56*, 37-43. https://doi.org/10.1007/S11204-019-09566-X