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STYLES OF MATHEMATICAL PROVING IN EAST AND WEST: THEIR USE IN TEACHING

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Abstract

In this paper, based on a conceptualization of the art of mathematical proof by means of the concept of style of reasoning, we can define different typologies of styles, viewed as outcomes of proof-events, depending on various measures. If for measure we take the admissible modes of reasoning, then three major types of styles (proof-events) can be defined: visual, constructive, and axiomatic proof-events (styles of reasoning), which have played an important role in history of mathematics. Moreover, they are commonly correlated with certain Western or Eastern mathematical traditions. Visualization-based proof events are associated with the Pythagorean pebble-arithmetic practice, Bhāskara's geometric proof of the Pythagorean Theorem, and others. Genetic or algorithmic kind of reasoning is associated with the Eastern mathematical traditions (Mesopotamian, Egyptian, Arabic, Indian, and Chinese), Brouwer's intuitionistic mathematics and Hilbert's finitary meta-mathematics. Axiomatic reasoning is usually associated with the Greek mathematical tradition, notably Euclid's Elements and Hilbert's new axiomatic construction of geometry in 1889. In the mid-1930's, the famous Bourbaki group, defined the new standards of axiomatic thinking, so that the types of proving by visualization and construction were displaced. These styles of reasoning can be used in intercultural mathematics education. The styles of reasoning of different agents (pupils / students) can be modelled as agents' profiles of a multi-agent system. Using different measures, we can get different typologies of proof-events (styles), and, furthermore different agents' learning profiles of the multi-agent system that can incorporate different cultural aspects. This can enhance the meta-cognitive skills in mathematical problem solving.

Keywords: Proof, styles of reasoning, intercultural mathematics.

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1. Introduction

Can we talk about the art of proof in mathematics? Mathematicians usually qualify proofs by such terms as:

Elegant proof (or formula or theorem),

Ugly proof (or formula or theorem),

Clumsy proof (or formula or theorem),

Awkward proof (or formula or theorem).

These qualifications have an aesthetic connotation; they are related with what is called beauty in mathematics. Thus, many mathematicians have attempted to describe mathematics as a form of art or, at a least, as a creative activity producing aesthetically assessable outcomes.

2. Problem Statement

How the art of proof can be conceptualized? As principal indicator has been suggested the concept of style of proof. A style can be:

personal for a mathematician, or

for the school he belongs, or

for a whole tradition.

It may be also mimicry of the style of a renowned authority.

3. Research Questions

Accordingly, certain questions arise:

What is style of proof after all?

Can style of proof be defined?

Can it be used in mathematics education? How?

In order to answer these questions, we appeal to a new concept: the concept of proof event (Goguen, 2001) and approach proof as a social process that evolves in time (Stefaneas & Vandoulakis 2014).

4. Purpose of the Study

The purpose of our study is to show that the possible correlations between styles of proving that have been developed in history and their cultural aspects, can been used in mathematics education, by taking into account the profile of the pupils/students. Typologies of styles of proving can be elaborated and taken into account in preparing culturally responsive mathematics curricula, as well as in teaching practice, particularly in cooperative distributive mathematical learning.

5. Research Methods

The current analysis is based on the following major areas of research:

The theory of proof-events, advanced in (Stefaneas & Vandoulakis, 2014, 2015b).

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Stefaneas and Vandoulakis' concept of style of proving (Stefaneas & Vandoulakis, 2014, 2015a; Vandoulakis & Stefaneas 2014; Vandoulakis, 2009).

The analysis of Web-based proof-events, in a sense similar to case of the Polymath project (Stefaneas & Vandoulakis, 2012).

Intercultural mathematics (Vandoulakis & Stefaneas, 2013a, 2013b; Vandoulakis & Liu, 2002).

6. Findings

Proof-events can be viewed as cooperative problem-solving activity of a multi-agent system evolving in space and time that generate proofs articulated in various semiotic codes and communicated in different styles (Vandoulakis & Stefaneas 2016). For multi-agent system can be taken a physical or virtual class with pupils / students, while the teacher / professor can enact the role of supervisor and ultimate validator of the proving outcomes.

A proof-event is initiated by a problem (defined by certain conditions). If we define "conditions" in a broad sense, as the admissible modes of reasoning over mathematical objects, then we can distinguish the following types of proof-events (styles) that have played significant role in history of mathematics:

Visualization-based proof-events;

Construction-based (genetic) proof-events, and

Assumption-based (axiomatic) proof-events.

In visualization-based proof-events, certain visual patterns or forms (proof objects) are used to convey mathematical ideas or a purported proof. The visual kind of reasoning conducted in such proof-events starts from a finite (usually small) domain of initial objects that are assumed as given beforehand, along with a set of operations, dependent on an associated concrete model (representation, visual image, configuration, graphical demonstration) for the propositions. The proof objects used in visualization-based proof-events have substantially changed throughout history, beginning from simple objects or signs (in antiquity) to more advanced computer-generated imageries generated by interactive tools (in the second half of the 20th century). Proof-events of this type probably go back to the Pythagorean pebble-arithmetic practice, as is described in the works of the Neo-Pythagorean authors, who expounded in writing the earlier non-linguistic Pythagorean tradition (Vandoulakis, 2009).

In construction-based (genetic) proof-events, certain (real or mental) constructions are performed in support of a purported proof. The constructive mode of reasoning conducted in such proof-events starts from a set of initial objects that are assumed as given beforehand, together with a set of admissible operations (e.g. algorithms, inductive definitions, etc.) over these objects. Reasoning is conducted over these objects and proceeds by construction of new objects out of the given ones by means of the admissible operations. Historically, the genetic or algorithmic kind of reasoning is associated with the Eastern mathematical traditions (Mesopotamian, Egyptian, Arabic, Indian, and Chinese). Computational algorithms abound in the Eastern mathematical literature, contrary to their limited presence in the Greek mathematical tradition. During the foundational debates of the early 20th century, constructive reasoning acquired the privilege of most reliable kind of proof. Luitzen Egbertus Jan Brouwer (1881-1966) allowed only constructive proofs in his intuitionistic mathematics, while David Hilbert (1862-1943) allowed only finitist methods in a consistency proof of mathematics. In assumption-based (axiomatic) proof-events,

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logical deductions from assumptions are used to justify a purported proof. The axiomatic mode of reasoning conducted in such proof-events starts from a relatively small set of propositions (called "(primitive) assumptions" or "axioms") that are assumed as given beforehand and taken for granted; these propositions are considered as describing a (generally infinite) domain (genus) of objects. Moreover, a set of logical operations (rules of inference) is given, the application of which generate new propositions that in turn describe new states of affairs in the domain. Axiomatic reasoning is usually associated with the Greek mathematical tradition, in particular Euclid's axiomatic organization of geometry in the Elements. The Euclidean axiomatic ideal remained unchanged until the 19th century. Lack of any attempt to axiomatize geometry in non-European mathematical traditions has reinforced the viewpoint that axiomatic mode of reasoning is essentially a European phenomenon. The shaping of the new era of axiomatic thinking is also related with Europe. It is related with the appearance of Lobachevsky-Bolyai's alternative geometry. This tendency is culminated in the new axiomatic construction of geometry in Hilbert's Grundlagen der Geometrie (1889), which serves as a historical landmark in the formation of the new concept of axiomatic thinking. In the mid-1930's, the famous Bourbaki group, undertook the task of rewriting the Éléments de Mathématiques (with a clear connotation to Euclid's Elements) by using the new standards of axiomatic thinking and displacing the types of proving by visualization and construction.

Based on the analysis of Polymath Web-based proof-event (Stefaneas & Vandoulakis 2012), we suggested a model for mathematics education, where mathematical learning is viewed as cooperative distributive discovery proof-event (Vandoulakis, 2016). This (sequence of) proof-event(s) can take place in a physical environment, i.e. in the classroom, or in a virtual environment, i.e. in a virtual class. The styles of reasoning of the different agents (pupils / students) can be modelled as agents' profiles of the multi-agent system under consideration. Using different measures (definitions of the "conditions"), we can get different typologies of proof-events (thereby different types of styles), and, furthermore different agents' learning profiles of the multi-agent system. These styles (profiles) incorporate different cultural aspects that underlie proof-events, but also can integrate different learning styles of the agents. This can enhance the meta-cognitive skills in mathematical problem solving of the agents.

7. Conclusion

Consequently, using the broader concept of "proof-event", instead of the concept of proof, we can define the concept of style of proof. This enables us to elaborate various typologies of styles of proof (proof-events), as well as various types of learning styles. In this way, we can integrate intercultural history of mathematics in mathematics education, notably in student-centred cooperative distributive learning.

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