

**CIEDR 2018**  
**The International Scientific and Practical Conference**  
**"Contemporary Issues of Economic Development of Russia:**  
**Challenges and Opportunities"**

**ON THE DYNAMICS OF DIFFERENT QUALIFICATION LABOR**  
**IN MACROECONOMIC ASPECT**

A. V. Prasolov (a)\*, A. A. Lukina (a), E. P. Dukhno (a)

\*Corresponding author

(a) Saint Petersburg State University, Universitetskay nab., 7/9, Saint Petersburg, Russia, a.prasolov@spbu.ru

*Abstract*

The task of a sustainable economic growth is impossible without considering the quality of labour factor. To consider it in a very simple way means to distinguish qualified and unqualified labour. The purpose of this report is to build a mathematical model of the dynamics of volumes of skilled and unskilled labour types. The main idea of the model consists in ascertaining the fact that any enterprise (an economic unit) tends to shorten the disbalance between mentioned above types of labour when this disbalance occurs. As this happens in a huge number of cases, then there is no point in considering each case separately. In this work, the algorithm to estimate a conditional balance (a synergetic balance) between these groups of labour in macroeconomic aspect is offered. The balance uncertainty is formed by different technological conditions of enterprises, dissimilar provision of labour groups in regions, different educational structures, nonstationarity of demography and other socio-economic factors. The statistical observations for the considered correlation are not conducted fully. However, production functions with different contribution of different labour groups appear in the scientific publication. So the mathematical modelling of the dynamics of these groups' volumes is of current importance.

© 2019 Published by Future Academy [www.FutureAcademy.org](http://www.FutureAcademy.org).UK

**Keywords:** Dynamic models, management of labour migration, skilled and unskilled labour, synergetic balance.



## 1. Introduction

In this work, we are trying to formulate a mathematical model of mutual dynamics of highly qualified and poorly qualified labour in relation with the attraction of labour migration. In the corresponding economic literature numerous definitions through payment for work, complication of given tasks, in connection with education etc. have been given (Strumilin, 1982). As a measure of qualification we offer the time of job training to consider (Bykova, 2011). Nearly every citizen in Russia has a secondary education, so an unskilled worker must differ from others by a complete absence of vocational education on the considered specialty.

“Workers” are considered to be all people able to work at the age from 15 to 63 (this is according to the Russian literature. According to the methodology of International Labour Organization, the number includes citizens from 10 to 72 years old (Internet-resource, 2018). In the work by (Kapelushnikov, 2006) economically active population includes people from 15 to 72 years old. We need factual numbers only for example, in order to understand how the algorithm works but not to forecast economic phenomena. So in the work we use rough but plausible values. Without migration, there are variable volumes of unskilled labour in every country. We enumerate some of them in order to estimate Russian reality quantitatively (here we use only evaluative quantities).

- a. Time between a secondary school and military service or university (this concerns both boys and girls) {1 mln} 1.
- b. Prisons {1 mln}.
- c. Army, National Guards and other enforcement agencies {Total number 4 mln, including 2 mln soldiers}
- d. Countryside {altogether 36 mln., 3 mln unskilled workers}
- e. Building industry {Total number 2 mln, 400 000 – unskilled workers}
- f. Homeless people {1 mln }
- g. Services of “shadow business” {Total 40% of working people in the economy, 10 mln – unskilled workers}
- h. Chemistry, coal, crude oil, gold, hunting, wood etc. {10 mln unskilled workers}.

We have altogether more than 20 million unskilled workers from the all 80 million working people. This is very rough, but it gives the following estimation 75% qualified and 25% unqualified labour.

To be more exact, from the work by (Kapelushnikov, 2006) and some other extrapolations we will obtain the following table with data on the volumes of skilled and unskilled labour (more exactly unskilled plus semi-skilled) for three years: in 1997, in 2002 and in 2007.

**Table 01.** Approximate data on the volumes of skilled and unskilled labour

	1997	2002	2007
Skilled labour (mln. people)	48	48.9	53.8
Unskilled labour (mln. people)	20.6	17.2	17

<sup>1</sup> In figure brackets we give rough estimation of workers which was taken from different sources on the Internet. The exactness in our argumentation does not play any role hence we do not forecast but we present a new mathematical model.

Further, in the example we will use these numbers. Summing up the majority of approaches to define a skilled worker, we obtain that a skilled worker is a person who makes decisions in the process of their activity and employers trust them to work with complicated and expensive elements K (basic assets, capital). A skilled worker bears legal and financial responsibility for mistakes in their activity.

An unskilled worker is a person who fulfils simple instructions in their activity and employers do not trust them complicated and expensive elements K. We consider that skilled and unskilled workers represent all possible working people in the country. In the context of the examined model and according to our definition from the great number of immigrants bus-drivers, taxi-drivers, drivers of building vehicles and etc. who received drivers' license legally are referred to skilled workers as they fulfil responsible work with people and machines every day. An important and biggest part of skilled workers among immigrants is tradesmen-owners as they professionally deal with management, marketing and contacts with authorities. To the group of unskilled workers we refer builders of simple operations, road builders, agricultural workers, janitors, cleaners, shop assistants, workers of sewing workshops (who are not owners) etc.

Let us partly describe how an unskilled worker transfers to the group of skilled workers. Firstly, it happens through general and professional education, retraining and other similar procedures (Breen & Goldthorpe, 1997). They can be paid, on the budget basis or in some other forms. Secondly, an unskilled worker can be motivated by career progress, financial family security, participation in business, social status etc. We cannot definitely avoid the possibility of cheating and bribery. It is impossible to take into consideration all driving forces in relation to every worker.

Let us consider how a skilled worker transfers to the group of unskilled workers. It is a more randomized problem, which is also hard to formalize. Some approach to description of workers movements between skilled and unskilled jobs is developed, for instance, in the search and matching theory (Mortensen, 1970; Lucas & Prescott, 1974; Pissarides, 2000, Acemoglu, 2001). In this literature the problem is treated exactly as the random one. Skilled workers concentrate on specialties, regions, economic and social sectors. Their destinies (a family destiny or a social group destiny) depend on technological, political, ecological and others changes. Bankruptcies, appearance of new technologies force people to change their qualification, possibly making them transfer into the group of unskilled workers. Requalification means the existence of a retraining system and this requires finances. If it is not a great problem from socio-political point of view, then a worker can change a place of living, a region and such things. Besides, there are diseases having which a person cannot get a disability status, i.e. a worker is not able to fulfil qualified work but able to work. One more reason to transfer to the group of unskilled workers is the surplus of workers with a certain specialty or qualification. In this case the absence of competitive advantages stimulates transition.

Here we come to the main point of mathematical modelling of a considered problem. The decision to transfer from one group into another is made by a worker but motivation to transfer is created by business or government. Instruments are competitive salaries, career prospects, working regime, dwelling privileges and such things. It is impossible to describe all the possibilities, which we considered and which can be added, with the help of one model. Therefore, to substantiate a model we think that in every

specific case the economy tends to create motivating conditions, which decrease discomfort of manufactures and workers as well.

In the country without immigration (if we do not take into account demography changes) there exists the balance between skilled and unskilled workers. This balance is determined by a technological level of a specific branch and a place (region, remoteness from consumers, from resources of raw materials etc.). The correlation is regulated by a salary: the attraction of a necessary number of skilled and unskilled workers is motivated by money (Harris & Todaro, 1970; Robinson & Tomes, 1982; Aghion & Blanchard, 1994), providing accommodation, benefits and other perks. Skilled versus unskilled labour choice is affected by domestic government policies that in turn can be affected by some globalization processes (Ogawa & Tsubuku, 2017). The appearance of unbalance bears a random character. So for short intervals of time, for instance 5 years, we offer to define the correlation statistically constant.

Nizhegorodtsev (2014) has written about an unsteady and non-linear character of the considered here economic parameters in; however, his argumentations are not given in the mathematical form.

If to take into account international immigration, then the problem under consideration becomes even more complicated. The skill composition of immigrants affects not only the skill composition of the entire population, but also economic growth, processes of human capital formation inside the recipient country, income growth and distribution etc. (Smith, 2012; Bandopadhyay & Chaudhuri, 2013; Ehrlich & Kim, 2015).

Naturally, the sum of workers from both groups is not constant: the general number of able to work citizens is inconstant because of demography oscillations, changes in the territory of the country, also economic needs in workforce. Essentially, it is these needs that call up immigrants or vice-a-versa push out workforce (for example, Latvia, Ukraine). It is considered that digitization will sharply decrease the number of working places for highly qualified people. The determined above balance is reached in a natural way as a certain limit of economy possibilities to bear expenses involved in the maintenance of unbalanced correlation between groups with different qualifications. This work is devoted to algorithms that allow to attract exactly so many immigrants how many is necessary to reach the balance.

## 2. Problem Statement

### 2.1. The task without immigration

We will introduce:  $S_b, U_b, S_m, U_m$  - skilled and unskilled native workers and skilled and unskilled immigrants respectively. We will think that in the state of balance there will be no big changes in the economy (few bankruptcies happen, few new working places appear, none new technologies appear etc.). In this case  $S_b = \bar{S}, U_b = \bar{U}$ .

Hence immigrants are not considered in this paragraph, then  $S_m = U_m = 0$ .

Bankruptcies, modernization and such things decrease the number of working places, which leads to the inequality  $S_b > \bar{S}$ , which returns to the equation  $S_b = \bar{S}$ , if changes are not big or have a random character. Stabilization in fact happens at the expense of redistribution of skilled workers into other regions, other professions or in the group of unskilled workers. The reason for such transformation is the decrease in salaries. If  $S_b < \bar{S}$ , then material stimulation is also used to compensate missing skilled

workers. This stimulation attracts specialists from other professions, other regions and what is more important encourages unskilled workers to train.

Thus the conditions to reach the balance:  $S_b \rightarrow \bar{S}$ , are created. The very simple model, which demonstrates the considered dynamics in time, is formalized as the ordinary linear differential equation:

$$\dot{S}_b(t) = r_s(\bar{S} - S_b(t)). \quad (1)$$

At positiveness of  $r_s$  there is  $S_b \rightarrow \bar{S}$ , i.e. asymptotic stability of equilibrium state.

For the group of unskilled workers we can formulate the same reasons for the aspiration of group volume for the balance. If  $U_b > \bar{U}$  (this is possible at demography boom, at closing of enterprises, at destruction of vocational education etc.), then wages reduce sharply. Working people go to earn money to other regions, enroll in the army, police, national guards, emergency control ministry and other state services. Some workers are employed to do heavy and cheap work, which leads to the degradation, to the lower part of a social ladder.

If  $U_b < \bar{U}$  (and this happens at a sudden appearance of the demand on  $U_b$ : building, seasonal agricultural work, epidemics and etc.), then employers increase wages for unskilled workers and this attracts workers from a skilled group or from other regions. The simplest model with attracting position of equilibrium, similar to (1), has the form

$$\dot{U}_b(t) = r_U(\bar{U} - U_b(t)). \quad (2)$$

In models (1) and (2) it is necessary to identify positive parameters  $r_s, \bar{S}, r_U, \bar{U}$ . In the simplest case for five given observations  $\{S_b(i)\}, \{U_b(i)\}, i=1,2,\dots,5$ , one can estimate parameters by mean values:

$$\bar{S} \approx \frac{1}{5} \sum_{i=1}^5 S_b(i); \quad \bar{U} \approx \frac{1}{5} \sum_{i=1}^5 U_b(i); \quad (3)$$

$$r_s \approx \frac{1}{4} \sum_{i=1}^4 \frac{S_b(i) - S_b(i+1)}{S_b(i) - \bar{S}}; \quad r_U \approx \frac{1}{4} \sum_{i=1}^4 \frac{U_b(i) - U_b(i+1)}{U_b(i) - \bar{U}}.$$

The built model (1)-(2) is good by its simplicity, but it does not show the interaction between  $S_b$  and  $U_b$ . Economically this interaction definitely exists. It is expressed through the increase of salaries and wages when there are not enough workers and the decrease of earnings when there are too many workers. We do not talk about changes  $\bar{S}$  and  $\bar{U}$  now as we assume that technologically, socially and legally economy changes insignificantly.

The development of the model (1)-(2) will be the consideration of the linear differential system:

$$\begin{cases} \dot{S}_b(t) = r_{SS}(\bar{S} - S_b(t)) + r_{SU}(\bar{U} - U_b(t)), \\ \dot{U}_b(t) = r_{US}(\bar{S} - S_b(t)) + r_{UU}(\bar{U} - U_b(t)). \end{cases} \quad (4)$$

The system (4) apparently has the only position of equilibrium  $(\bar{S}, \bar{U})$  and it is asymptotically stable when  $r_{SS}, r_{UU} > 0$ , and  $r_{US} r_{SU} < 0$ .

The parameters  $r_{US}$  and  $r_{SU}$  are responsible for the interaction between the groups of workers creating oscillation of the process.

## 2.2. About identification of the system (4)

Unknown parameters are  $r_{US}, r_{SU}, r_{SS}, r_{UU}, \bar{S}, \bar{U}$ . They come into the system nonlinearly. So we cannot obtain them in a standard way. We can apply the algorithm of simultaneous identification of system parameters and delay from the works by (Prasolov, 2016; Prasolov, 2018). Briefly it lies in parameters  $\bar{S}, \bar{U}$  fixation and the optimal selection of coefficients  $r_{US}, r_{SU}, r_{SS}, r_{UU}$ , and then the procedure is repeated by reselection of parameters  $\bar{S}, \bar{U}$ . The algorithm comes to a stop at the best set of parameters in some sense. Let us have  $n$  observation of values  $S_b$  and  $U_b$ . Using the formula (3) we will estimate the parameters  $\bar{S}, \bar{U}$ :

$$\dots\dots \bar{S} \approx \frac{1}{n} \sum_{i=1}^n S_b(i); \quad \bar{U} \approx \frac{1}{n} \sum_{i=1}^n U_b(i) \dots\dots\dots (5)$$

In the system (4) we will change the variables:  $x = S - \bar{S}, y = U - \bar{U}$ . Then the system (4) will be as follows

$$\frac{d}{dt} \begin{pmatrix} x \\ y \end{pmatrix} = -R \begin{pmatrix} x \\ y \end{pmatrix}, \quad R = \begin{pmatrix} r_{SS} & r_{SU} \\ r_{US} & r_{UU} \end{pmatrix}.$$

The solution to Cauchy problem for this system is written through the matrix exponent:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \exp(-Rt) \begin{pmatrix} x(0) \\ y(0) \end{pmatrix}.$$

If we accept the observation interval equal to 1 (for example 1 year), then we have the set of numbers  $\{x(i)\}, \{y(i)\}, i = 1, \dots, n$ ; and the approximate equation holds true

$$\begin{pmatrix} x(i+1) \\ y(i+1) \end{pmatrix} \approx \exp(-R) \begin{pmatrix} x(i) \\ y(i) \end{pmatrix}. \quad (6)$$

Using all the accessible for us observations, we will obtain the estimation of a matrix exponent as the mean of some matrixes:

$$\exp(-R) \approx \frac{1}{n-2} \sum_{i=1}^{n-2} \begin{pmatrix} x(i+1) & x(i+2) \\ y(i+1) & y(i+2) \end{pmatrix} \begin{pmatrix} x(i) & x(i+1) \\ y(i) & y(i+1) \end{pmatrix}^{-1}. \quad (7)$$

**Note:** The systems of differentials equations are brought to discrete time by Euler method, but in this case when the system is linear the offered algorithm is more exact and quicker. If some matrixes participating in the last formula turn out to be degenerated, this combination of vectors one can delete from the mean.

## 2.3. The example

In Table 01 we give estimation observations for the volumes of labour during the interval of 5 years. If we consider this period as a time unit, we can show how to build a dynamic model using the formulae (6) and (7) with the following forecast (for 2012 year). Three columns in the table we will designate in a more convenient way:

$$\begin{pmatrix} S(1997) \\ U(1997) \end{pmatrix}, \begin{pmatrix} S(2002) \\ U(2002) \end{pmatrix}, \begin{pmatrix} S(2007) \\ U(2007) \end{pmatrix},$$

where the relevant year is put in as an argument. We will find the mean value  $\bar{S}, \bar{U}$ . using the formulae (5): Then using the formula (7) we will obtain the matrix

$$\exp(-R) \approx \begin{pmatrix} S(2002) - \bar{S} & S(2007) - \bar{S} \\ U(2002) - \bar{U} & U(2007) - \bar{U} \end{pmatrix} \begin{pmatrix} S(1997) - \bar{S} & S(2002) - \bar{S} \\ U(1997) - \bar{U} & U(2002) - \bar{U} \end{pmatrix}^{-1}$$

The result of modelling is shown with the formula

$$\begin{pmatrix} S(2012) \\ U(2012) \end{pmatrix} \approx \exp(-R) \begin{pmatrix} S(2007) \\ U(2007) \end{pmatrix} + \begin{pmatrix} \bar{S} \\ \bar{U} \end{pmatrix}.$$

After substituting the values of estimation data from the table, we will estimate the forecast of labour volumes with numbers

$$\begin{pmatrix} S(2012) \\ U(2012) \end{pmatrix} \approx \begin{pmatrix} 48 \\ 20.6 \end{pmatrix}.$$

The result predictably has coincided with the first column in the table 01 as we compensated the absence of data by linear extrapolation. Nevertheless, the example shows the possibilities of the method.

The building of matrix logarithm, i.e. the matrix (-R) is well-known, for example (Gantmacher, 1959; Prasolov, 2016). In particular, if matrix's (-R) numbers lie in the right half-plane of the complex plane, then we can use the following series expansion

$$-R = \sum_{l=1}^{\infty} \frac{(-1)^{l-1}}{l} (\exp(-R) - E)^l.$$

### 3. Research Questions

#### The appearance of labour migration

The appearance of immigrants disrupts the balance: firstly, adding immigrant skilled workers to native skilled workers creates competition and forces out the part of skilled workers into the group of unskilled workers. This competition we can observe in transport sphere, building industry and small business etc. Secondly, unskilled workers actively occupy working places in unpopular activities (low-paid, dirty, risky, monotonous and such types of jobs. This allows (and sometimes forces) local population to improve their qualification and to transfer to the group of skilled workers. Besides, some immigrants gradually come to feel at home, are trained and join the skilled workers' group.

Let migration take place in the volume M during the year. Meanwhile it is not important if immigrants are attracted by high productivity (Lukina & Prasolov, 2016) or by higher level of consumerism in the country. Formally, we will consider M as the endogenous parameter. Moreover, it is difficult to distinguish skilled and unskilled parts as immigrants come from different countries which have different vocational systems of education and different technological levels of economy.

In accordance with the designation  $S_m, U_m$  we assume that  $U_m = \sigma S_m$ , where the parameter  $\sigma$  is defined statistically. Then

$$M = S_m + U_m = (1 + \sigma)S_m.$$

The model (4) with known coefficients is transformed into

$$\begin{cases} \dot{S}_b(t) = r_{SS}(\bar{S} - S_b(t)) + r_{SU}(\bar{U} - U_b(t)) + \frac{M}{1+\sigma}, \\ \dot{U}_b(t) = r_{US}(\bar{S} - S_b(t)) + r_{UU}(\bar{U} - U_b(t)) + \frac{\sigma M}{1+\sigma}. \end{cases} \quad (8)$$

Now we can set the task of management with the help of M. There are many variants!

We note that at  $M \neq 0$  the system (8) has the position of equilibrium  $(\bar{S}, \bar{U})$  different from  $(\bar{S}, \bar{U})$ . It is comparatively easy to find it: we solve the system

$$\begin{cases} 0 = r_{SS}(\bar{S} - \bar{S}) + r_{SU}(\bar{U} - \bar{U}) + \frac{M}{1 + \sigma}, \\ 0 = r_{US}(\bar{S} - \bar{S}) + r_{UU}(\bar{U} - \bar{U}) + \frac{\sigma M}{1 + \sigma}. \end{cases}$$

We will obtain  $\bar{S} = \bar{S}(M)$  and  $\bar{U} = \bar{U}(M)$ , and these are linear functions of the scalar argument M. That is we will have the balance in economy at different volumes of migration:

$$\begin{pmatrix} \bar{S} \\ \bar{U} \end{pmatrix} = \begin{pmatrix} \bar{S} \\ \bar{U} \end{pmatrix} + \frac{M}{1 + \sigma} R^{-1} \begin{pmatrix} 1 \\ \sigma \end{pmatrix}.$$

**Note.** In this formula  $(\bar{S}, \bar{U})$  were taken very approximately,  $R^{-1}$  depends on this selection. Therefore, obtained  $(\bar{S}, \bar{U})$  are also approximation. To have a more precise level of the balance we can use several iterations.

## 4. Purpose of the Study

### Management tasks

**The task A.** To find M at which the disbalance of economy is minimal:

$$M_A = \arg \min \left[ \left( S_B(T) - \bar{S}(M) \right)^2 + \left( U_B(T) - \bar{U}(M) \right)^2 \right].$$

This task is important for defining the state of economy at the current moment T. The equation in the square brackets shows how big the disbalance is and if it is possible to return the balance with the help of labor migration.

*The task B.* We assume that the outcome of economy, i.e. GDP, is determined by the production function  $Y = AK^\alpha (\bar{S}(M)^\nu \bar{U}(M)^{1-\nu})^{1-\alpha}$  (Dmitriev & Yudina, 2017). The task is to find M at which the growth of GDP is maximal:

$$M_B = \arg \max (AK^\alpha (\bar{S}(M)^\nu \bar{U}(M)^{1-\nu})^{1-\alpha}).$$

We can consider the problem of the given growth at minimal M.

*The task C.* We assume that the growth of capital is determined by the differential equation  $\dot{K} = -\delta K + \rho(1 - a)Y$  (Dmitriev & Yudina, 2017).

To find M at which the capital growth K will be maximal:

$$M_C = \arg \max (-\delta K + \rho(1 - a)Y).$$

Due to the homogeneity of the production function on labour all the three tasks can be solved analytically as M in them is a scalar value and enters linearly.

All these tasks are united by the necessity to analyze if we can find the given M at different assumption of its appearance. If there is no labour migration, we consider the forecast in accordance with demography dynamics. If there is labour migration, we consider together the change in number of workers and plus migration. Then we include the system of quotas setting.



## 5. Research Methods

We built the system of differential equations in its simplest form: the linear form. The sense of attraction to the balance lies in the fact that enterprises of economy create motivation for workers to occupy working places or to leave them depending on technological conditions of an enterprise. Thus, the system equilibrium is non-stationary in the long-time interval, it is immeasurable, in principle. This is why, an adaptive method of construction, which uses the previous state of economy, is offered.

## 6. Findings

We have shown how to use the introduced balance in the tasks of migration flow management. We have formulated three management tasks, which use the built mathematical model. At the availability of real data these tasks can be applied to determine quota for labour migration flows, which compiles the purpose to take quantitative decisions by regional and federal authorities.

## 7. Conclusion

The task of labour migration management reaches the new level of adequacy, if we take into account the difference in contribution of workers with different qualification. In this work we have fulfilled an important stage of modelling: we have offered the algorithm of dynamics of skilled and unskilled labour volumes. We have shown some further steps based on taking into consideration demography and the contribution of differently qualified workers. The main idea of the algorithm consists in the assumption that economy tends to a certain synergetic balance, creating motivation for each enterprise from its own recourses and this motivation integrally forms the whole economic tendency. Macro-economically it is not possible to determine the number of skilled and unskilled workers, so we introduce the described above conditional notion - synergetic balance. Analogies were repeatedly encountered in science: for example, pressure in gas, absolute temperature and other notions of statistical physics. When there are enough empirical data, the offered model can be successfully used for making macro-economical decisions.

## Acknowledgments

The report was supported partly by RFBR № 18-01-00551 (the author (b)). We appreciate it. Also this article was prepared in part while the author (b) held a postdoctoral fellowship with the Wisconsin Russia Project, sponsored by Carnegie Corporation of New York.

## References

- Acemoglu, D. (2001). Good Jobs versus Bad Jobs. *Journal of Labor Economics*, 19(1), 1–21.
- Aghion, P., & Blanchard, O.J. (1994). On the Speed of Transition in Central Europe. *NBER Macroeconomics Annual*, 9, 283–320.
- Bandopadhyay, T. K., & Chaudhuri, S. (2013). Endogenous Skill Formation: Labor Market Integration by an Immigration. *Journal of Economic Integration*, 28(1), 183-200.
- Breen, R., & Goldthorpe, J.H. (1997). Explaining Educational Differentials. Towards a Formal Rational Action Theory. *Rationality and Society*, 9(3), 275-305.

- Bykova E. A. (2011) [in Russian]. Education role in labour factor quality formulation in modern Russia. *Living standard of Russian regions*, 2 (156), 47-51.
- Dmitriev, M.G., & Yudina, T.N. (2017) [in Russian]. Migration processes; models of analysis and forecasting (review). *Proceedings of System analysis institute of Russian Academy of Science*, 67(2), 3-14.
- Ehrlich, I., & Kim, J. (2015). Immigration, Human Capital Formation and Endogenous Economic Growth. *Journal of Human Capital*, 9(4), 518-563.
- Harris, J. R., & Todaro, M. P. (1970). Migration, Unemployment and Development: A Two-Sector Analysis. *The American Economic Review*, 60(1), 126-142.
- Gantmacher, F. R. (1959). *The theory of matrices*. New York: Chelsea Publishing Company.
- Internet-resource (2018). Received July 24, 2018, from <https://biznes-prost.ru/ekonomicheskii-aktivnoe-naselenie.html>
- Kapeliushnikov, R. I. (2006). *Russian labour structure: features and dynamics*. Moscow: HES.
- Lucas, R., & Prescott, E. (1974). Equilibrium Search and Unemployment. *Journal of Economic Theory*, 7(2), 188-209.
- Lukina, A.A., & Prasolov, A.V. (2016). The Labour Immigration Control. *International Journal of Pure and Applied Mathematics*, 108(3), 659-669.
- Nizhegorodtsev, R. M. (2014). Labour migration as a tool of human capital quality update. *Izvestia of the Ural economic state university*, 3(53), 6-10.
- Mortensen, D.T. (1970). Job Search, the Duration of Unemployment and the Phillips Curve. *The American Economic Review*, 60(5), 847-862.
- Ogawa, H., & Tsubuku, M. (2017). Impact of Capital Market Integration on Skilled-Unskilled Labor Choice. *Journal of Economic Integration*, 32(1), 90-111.
- Pissarides, C. A (2000). *Equilibrium Unemployment Theory*. The MIT Press. .
- Prasolov, A. V. (2016). *Some quantitative methods and models in economic theory*. NY.: Nova Science Publishers.
- Prasolov, A. V. (2018). On the simultaneous estimation of delay model parameters in economic dynamics. *Physica A: Statistical Mechanics and its Applications*, 509, 1102-1109.
- Robinson, C., & Tomes, N. (1982). Self-Selection and Interprovincial Migration in Canada. *The Canadian Journal of Economics*, 15(3), 474-502.
- Smith, C.L. (2012). The Impact of Low-Skilled Immigration on the Youth Labor Market. *Journal of Labor Economics*, 30(1), 55-89.
- Strumilin, S. G. (1982). *Labour economic problems*. Moscow: Nauka.