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CAMBRIDGE COMPROMISE AS A METHOD OF DEGRESSIVELY PROPORTIONAL DISTRIBUTION OF GOODS

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Abstract

One of generally accepted principles of allocating goods and obligations is the principle of proportionality. According to this principle, agents participating in division are allocated a part of goods or are required to incur a part of obligations in proportion to their values. The principle behaves well in practice if the values describing the entitlements of agents do not differ a lot. Otherwise, the smallest agents may be disregarded, what cannot be always agreed to because of the common interest of the group participating in allocation. This problem occurred in practice when the seats in the European Parliament were apportioned among the member states. Therefore, the Treaty of Lisbon (2007) includes the rule of degressively proportional division of seats in the European Parliament. According to this Treaty, the representation of member states should be degressively proportional with respect to the populations of respective members of the community. Such stipulations allow different interpretations and therefore, different results of parliamentary representation. One of the solutions presenting actual allocation of seats is the so-called Cambridge Compromise put forward by the team of experts working under the auspices of the Commission of the European Parliament. However, this proposition is not general and in its present form can be applied only to solve a single, given task. This paper presents a proposition to generalize the Cambridge Compromise, so that its idea can be applied as a method of allocation in any task of integer, degressively proportional distribution of indivisible goods

Keywords: Cambridge Compromise, European Parliament, degressive proportionality.

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1. Introduction

European culture is basically characterized by the underlying principle of fair distribution of benefits and obligations enabled by the principle of proportionality. According to it, each agent is allocated such amount of good or burdened by obligations shared in a degree equal to the share of its value in the total value of all agents. For example, in one of the fundamental practical issues, this principle stipulates that the number of representatives of should be proportional to the number of people living.

2. Problem Statement

In some situations, however, it is recommendable to abandon the principle of proportionality. In case of large disproportions in populations of respective constituencies, a proportional distribution can lead to marginalization of some regions, while political conditions usually do not allow such solutions. For example, a significant diversification of populations in member states of the European Union was one of the reasons why representation of respective countries in the European Parliament proportional to the number of inhabitants had to be abandoned. Given the total size of the assembly, in case of proportional distribution, Malta could have just one representative, and the Maltese would have no chances to effectively work in this collegiate body.

As an alternative solution, the Treaty of Lisbon (2007) put forward a degressively proportional rule of determining the parliamentary representation. The exact wording of the respective paragraph is following: "*The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats*". Trying to formalize this provision, one can write that a division *S* is degressively proportional with respect to *P* when

(1) for every $1 \le i < n$ it holds $s_i \le s_{i+1}$, and

(2) for every $1 \le i < n$ it holds $\frac{s_{i+1}}{p_{i+1}} \le \frac{s_i}{p_i}$,

where a sequence of positive numbers $P = (p_1, p_2, ..., p_n)$ is a sequence of populations of *n* member states, and a sequence of natural numbers $S = (s_1, s_2, ..., s_n)$ defines the number of seats allocated to them. Moreover, the cited provision also determines the total size of the European Parliament *H* and the so-called boundary conditions, i.e. a minimum and a maximum nuber of mandates allocated to a member state:

(3) $s_1 + s_2 + ... + s_n = H$,

(4) $s_1 = m = 6$, $s_n = M = 96$.

3. Research Questions

In a general case, one can certainly choose the values of *H*, *m* and *M* in such a way that, given the sequence *P,* there is no sequence *S* satisfying the conditions (1)–(4). Then, it is not possible to apply the rule of degressively proportional allocation of H goods with respect to boundary conditions m and M for such P. This demonstrates the need of appropriate determination of the boundary conditions of individual tasks to find a degressively proportional allocation (Łyko, 2012; Łyko, 2013; Dniestrzański & Łyko, 2014b). In case of adequate choice of these parameters, there could be many possible solutions, and one needs a clue to pick up one of them as an actual allocation.

4. Purpose of the Study

One of the best known and widely discussed proposals of degressively proportional division of seats in the European Parliament is the so-called Cambridge Compromise (Grimmett et al., 2011; Grimmett, 2012). This solution was elaborated as a result of a meeting in Cambridge organized by the Committee on Constitutional Affairs. It consists in allocating seats with the use of allocation function:

$$A(p) = \min\left\{ b + \frac{p}{d}, M \right\}$$

where parameter b is called the base and d is a free parameter called the divisor. Based on the function $A(p)$, the allocation is generated taking the value of the parameter b equal 5 and choosing the divisor d in such a way, that after rounding upwards the terms of the sequence $A(p_1)$, $A(p_2), \dots, A(p_n)$, their sum would equal H=751, i.e. the total size of the European Parliament. To be more precise, the following values are considered: $s_i = \min\left\{ \left\lceil b + \frac{p_i}{d} \right\rceil, M \right\}$, where M is here equal 96, and choice of the parameter b and the structure of the sequence $P = (p_1, p_2, \dots, p_n)$ guarantee that $s_1 = \min\left\{ \left\lceil b + \frac{p_1}{d} \right\rceil, M \right\} = m = 6$, i.e. the boundary conditions imposed by the Treaty of Lisbon (2007) are satisfied.

The Cambridge Compromise is an apparent compilation of two allocations: equal and proportional. The equal part is represented by the first element called base, and the proportional part by the second one, consistent with the Adams divisor method. As is known, the Adams method, among other divisor methods, predominantly favors smaller countries, thus implying that the authors of the Cambridge Compromise wanted to gain the effect of the largest representation of the smallest members of the community, with respect to conditions stipulated by the Treaty of Lisbon (2007) and the divisor method of allocating the seats distributed proportionally.

In this way, the generated allocation obviously meets the condition (1), but, due to rounding, it does not necessarily satisfy the condition (2) of degressive proportionality. Therefore, to be able to put forward a practical solution, the amendment was passed in 2007 (Lamassoure & Severin, 2007), modifying the wording of paragraph 6 to the following form: *[The European Parliament] Considers that the principle of degressive proportionality means that the ratio between the population and the number of seats of each Member State before rounding to whole numbers must vary in relation to their respective populations in such a way that each Member from a more populous Member State represents more citizens than each Member from a less populous Member State and conversely, but also that no less populous Member State has more seats than a more populous Member State.*

It is easy to demonstrate that for a given sequence of populations $P=(p_1,p_2,\ldots,p_n)$ the values of the function A on elements of this sequence, i.e. the numbers $A(p_1),A(p_2),\ldots,A(p_n)$ generate a degressively proportional sequence. Thus, for each $1\le i<n$ it holds $A(p_i)\le A(p_{i+1})$, and for each $1\le i<n$ it holds $A(p_{i+1})/p_{i+1}\le A(p_i)/p_i$, i.e. the condition from the resolution is satisfied and the Cambridge proposal has been legally endorsed. The literature (Ramírez González et al., 2012; Delgado-Márquez et al., 2013) adopted terminology of "unrounded degressive proportionality" (Słomczyński & Życzkowski, 2012; Cegiełka, 2016) in opposition to "rounded degressive proportionality", where allocation $S=(s_1,s_2,\ldots,s_n)$ satisfies the conditions (1) and (2). (Maciuk, 2011; Dniestrzański & Łyko, 2014a; Haman, 2017).

5. Research Methods

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6. Findings

First of all, let us notice that the set $B(P,H,m,M)$ of all nonnegative integers b satisfying the property that the sequence of terms $s_i=\min\left\{\left\lceil b+\frac{p_i}{d}\right\rceil,M\right\}$ is an allocation with respect to conditions $s_1=m$, $s_n=M$, of H goods among n agents, whose entitlements are represented by the sequence $P=(p_1,p_2,...,p_n)$, can be the empty set. In fact, it suffices to notice that such a case occurs when the values of P, H, m and M are as in example 1.

Example 1.

It is easy to notice that given the values $P=(10,11,12)$, $H=10$, $m=3$, $M=4$, the set $B(P,H,m,M)$ is empty.

Therefore, it is not always possible to apply the idea of the Cambridge Compromise in the sense of "unrounded" degressively proportional allocation, i.e. as a sum of one constant part and one proportional divisor part with rounding upwards.

If the set $B(P, H, m, M)$ is a one-element set, the idea of the Cambridge Compromise can be generalized without problems. The choice of the base is unique. Problems emerge when there are at least two elements in the set $B(P, H, m, M)$. Examples 2 and 3 demonstrate that in such a case adopting different values of the base can lead to the same or different allocations.

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Example 2.

Table 1 presents the case where different values of the base b result in the same division of goods. Input data are: $P = (13,17,40,40,60)$, $H = 22$, $m = 3$, $M = 6$, and two different bases: $b = 1$ and $b = 2$.

Table 1. Different values of the base generate the same allocation of goods

	***b* = 1**	***b* = 2**
p_i	s_i	s_i
13	3	3
17	3	3
40	5	5
40	5	5
60	6	6
Total: 170	Total: $H = 22$	Total: $H = 22$

Source: own elaboration.

Example 3.

Table 2 presents the case where different values of the base b generate different allocations of goods. Input data are: $P = (10,18,40,40,60)$, $H = 22$, $m = 3$, $M = 6$, and two different values of the base: $b = 1$ and $b = 2$.

Table 2. Different values of the base generate different allocations of goods

	***b* = 1**	***b* = 2**
p_i	s_i	s_i
10	3	3
18	3	4
40	6	5
40	6	5
60	6	6
Total: 170	Total: $H = 22$	Total: $H = 22$

Source: own elaboration.

Moreover, it is easy to notice that for any value of *m* one can choose such values of *P*, *H* and *M* that $B(P,H,m,M) = \{1,2,...,m-1\}$ holds. In other words, each number from the set $\{1,2,...,m-1\}$ can be the base. The suitable choice of parameters *P*, *H* and *M* can be as in example 4. This is really any possible value of the base, because assuming that elements of the sequence *P* are positive, taking *b*=*m* leads to the equality $s_1 = m+1$, after rounding upwards, therefore one of the boundary conditions is not satisfied. Clearly, this also demonstrates the boundedness of the set $B(P,H,m,M)$.

Example 4.

Let *m* be any positive integer, then taking $H = 2m + 1$, $M = m + 1$ and $P = (p_1, p_2)$, such that $p_1 < p_2$ and assuming as the base any number b from the set $\{1,2,...,m-1\}$, we obtain the allocation

$s_1 = m$, $s_2 = m+1$.

The authors of the Cambridge Compromise assumed $b = m - 1$ to solve one specific task of allocating the seats in the European Parliament. However, this approach cannot be generalized for other allocations. Example 5 demonstrates that $m - 1$ does not always is an element of the set $B(P,H,m,M)$, even if this set is not empty.

Example 5.

Given input $P = (1,2,4)$, $H = 10$, $m = 3$, $M = 4$, the number $b = m - 1 = 2$ does not belong to the set $B(P,H,m,M)$, although $B(P,H,m,M)$ is not empty, because one can easily notice that $1 \in B(P,H,m,M)$.

Hence, when generalizing the idea of allocation according to the Cambridge Compromise, one should assume that the parameter *b* is the greatest element of the set *B*(*P*, *H*, *m*, *M*). In view of the boundedness of this set as asserted above, this definition is evidently correct, and the Cambridge Compromise is a special case of this general method. Therefore, we can present this method as a generalization of a previously known single solution, that was put forward during the Cambridge Meeting. Because of the choice of the base and the way of rounding, this method of allocating goods with the sum of a constant element, i.e. the base, and the proportional element, obtained by divisor methods of proportional allocation, is most advantageous to smaller agents.

7. Conclusion

The allocation method proposed as a solution to a single problem of allocating the seats in the European Parliament among member states can be generalized for a case of any degressively proportional allocation. The generalization consists in indicating a general manner of selecting the so-called base, which was arbitrarily indicated by the authors of the Cambridge Compromise. We recommend taking the greatest integer for which there exists a degressively proportional allocation with respect to given boundary conditions. This generalization preserves a very significant property of the Cambridge Compromise, i.e. giving preference to agents with smaller claims.

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