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DENSITY AS A RATIO-CONCEPT: CULTURAL-HISTORICAL ACTIVITY APPROACH IN TEACHING

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Abstract

We discuss the potential of Cultural-Historical Theory (Vygotsky) and Activity Approach (Leontiev) for teaching ratio-based concepts (in particular – density) in middle school. The Activity Approach assumes that the concept should originate from some meaningful object-related action. The logical and activity content of the concept of density was analyzed. We have suggested the float-or-sink problem as the proper context for introducing density. Changing the object's buoyancy thus serves as students' meaningful activity that brings the density concept into consideration. We also used a specific stratagem for teaching ratio-based concepts: "nominator" and "denominator" (for density, these are weight and volume, respectively) were initially presented as independent features of an object measured in some artificial units. A local instruction theory (educational design research framework) has been devised; 42 students (6th grade) participated in our teaching experiment, while another 40 students (7th grade) were a control group where density has been studied within the 'problem-and-practice' physics curriculum. The pre- and post-test of our students were compared to that of the control group. The post-test results were analyzed in terms of strategies that students used, explanations given by the students, mistakes made, etc. The experimental group has shown significantly better results as compared to the control group regarding understanding of density as a ratio-based concept and dealing with it. The key to success, in our opinion, is that the Activity Approach stimulates educators to pay special attention to the operational content of the ratio-based concepts. This ensures much better learning outcomes.

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Keywords: Ratio-based concepts, "density, activity approach

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1. Introduction

Density is a concept used both in science and in everyday life. This concept is mathematically expressed as a ratio of two different values (mass and volume). Other ratio-concepts are: concentration, velocity, flow rate, pressure, power, etc. These concepts are known as difficult to assimilatefor students (Hecht et al., 2007; Kloosterman, 2010, etc.) There are indications that it is the presence of ratio that makes them hard to comprehend and apply in problem solving (Sophian, 2007).

For decades, psychologists and educators have studied how students assimilate these concepts and what can be done to improve instruction (from guided-inquiry based teaching (Almuntasheri et al., 2016; Moli et al., 2016) to development of students' argumentation (Chen et al., 2014)). New opportunities offered by digital era have also found their way into this part of curriculum (e.g. Çepni & Şahin, 2012, Stott & Hattingh, 2015). Most psychological studies focus on the strategies and rules that children use to predict the results, how the rules change according to age, feedback, possibility to perform some experiments, etc. (Jansen, van der Maas, 2002; Nunes et al., 2003). Other authors (Siegler, Chen, 2008, p.444). made a conclusion that teaching children to differentiate clearly between two parameters improved their ability to solve problems in each domain.

Educators are developing constructivist and sociocultural approaches to teaching math and science (e.g. Tobin, 1993; Fensham et al., 1995) because active and social nature of learning is obvious. The Vygotsky's conception that has already spread around the world has been elaborated by his collaborators and followers such as Leontiev (activity approach), Galperin (systematic formation of mental actions and concepts), Elkonin, Davydov (developmental education), Engeström (expansive learning), etc. Many interesting ideas have been suggested to improve instruction, and some of these insights come closely to the Vygotsky's conception about a sociocultural nature of learning. However, designing a perfect learning environment, instruction, and curriculum that are feasible is still a challenge.

2. Problem Statement

In the regular school curriculum, the concept of density is presented by its definition and formula and often used in connection to the explanation of the effect of buoyancy. Usually, the illustrations involve a couple of simple experiments that allow students to compare and contrast the behavior of objects made of different materials if put into water. The topic also includes the notion of the buoyant force and the Archimedes's law. Following that, the students are given a few problems that require simple calculations based on the formula of density or comparison of the objects' densities, and so on. Thus, students' actions with the concept are limited by its application to solving the problems that, in the best case, already involve an actual concept of density.

According to the 'problem-and-practice' curriculum for 6-graders (Lvovsky et al. 2009), the concept of density emerges through comparing the objects made of different materials but having equal volumes or weights. Students realize that the objects with equal weights may have different volumes, and objects with equal volumes may have different weights. Then students shift from two parameters (weight and volume) to one, which is density of the object presented as a ratio of its weight and volume. At this stage, students calculate density of different materials by measuring weights and volumes and checking the results against a reference book. At the next stage, students discover that an object immersed in water weighs less than in the air. They explore the buoyant force and Archimedes's principle by performing experiments and figure out what determines the magnitude of the buoyant force (volume of an object and liquid density). These two features determine buoyancy of an object (in other words – its ability to sink or float).Analysis of these teaching materials shows that the authors failed to consider the specific activity content of density as a concept of ratio; instead, they suggest forming this concept like any other by connecting two essential parameters in the students' minds, but not in their actions. It is a possible cause of students' difficulties in learning ratio-concepts that we address in the study.In so doing, we look for a specific content of the concept of density that helps students to start connecting two parameters in their actions.

3. Research Questions

As we implement the Activity approach to the practice of school education, we regard logical operations and other mental skills that are required to solve various problems in a domain as derivatives from certain meaningful object-related actions. The initial stage that leads to acquisition of a new action by children is interpsychological (term by Vygotsky) and occurs between people. In order to individualize it, a person has to perform the action in three consequent forms: material, verbal, and mental, which allows him to internalize the 'orienting basis of the action'. Scientific concepts are also considered to be derivatives from these goal-oriented actions (e.g. Galperin, 1989; Arievitch, Haenen, 2005; Rambusch, 2006; Zuckerman, 2003).

According to Davydov, a genuine concept is a general method of acting, i.e., a method for solving a large class of problems that is related to the entire system of object-oriented actions. Assimilation of the concept is associated with performing of a so-called 'transforming action' (Davydov, 2008; Engeström, Sannino, 2010; Ilyenkov, 1974; Moxhay, 2008). The concept presents all necessary features that require consideration (on the material level at first, and in mind subsequently) when ***the object is transformed according to the given purpose***. Thus, we assume, that the concept of density presents the factors to be taken into account if we want ***to keep or change*** the buoyancy of a 'vessel' (Vysotskaya, 1996).

To test our assumption a local instruction theory has been formulated within an educational design research framework (Van den Akker, Gravemeijer, et.al, 2006).It's features are as following:

- students are involved into a domain-specific 'transforming' action that allows students to accept the goal of the action and uncover the necessary action-orienting components to reach the goal,
- essential components are presented clearly, so the students are able to physically manipulate them and figure out how they work,
- the responsibility for each alterable parameter is divided between the students, so they need to coordinate their actions in a proper way to reach the goal, and, as a consequence, students can figure out how the parameters are related to each other;
- students are given a tool that allows them, at first, to record the actions and their results, and then, to plan the subsequent actions, so this tool serves as a model of the action and develops along with the action, eventually becoming a model of the phenomenon, i.e., the actual

concept. The role of this model is to coordinate the individual actions of the students in a pair and test their results in the practice. Thus, each contribution into a common result of the action can be represented explicitly, such that the necessary features of the concept are reconstructed for the students.

So, the main research question is as follows: Does our instruction provide a solid foundation for assimilation of the concept of density as a ratio concept? Is the content outlined above sufficient to establish students' understanding of this ratio concept?

4. Purpose of the Study

Our long-term goal is to specify the types of activities that provide a correct construction and comprehension of the concept of ratio in the teaching-learning process. In order to design a local instruction theory for the introduction of the concept of ratio, we exploited Davydov's approach, which is one of the most elaborated branches of the 'Activity Theory' and tightly binds the theory and educational practice (e.g. Dougherty, Slovin, 2004; Giest, Lompscher, 2003; Moxhay, 2008; Zuckerman, 2003). This approach requires to reconstruct the origin of a concept in material actions of students (Davydov, 2008).

Inwhat follows, we present a part of our work related to the concept of density. We describe a test aimed at assessing students' ability to solve problems that require this concept. Then, we outline the design of our experimental instructional module. Finally, we present the results of the post-instruction assessment and discuss them.

5. Research Methods

5.1. Assessments

To assess how the students assimilate the concept of density as a ratio concept, we select some problems that require from students: (a) to work with conservation of the ratio, and (b) to compare the ratio values when the values of both parameters are different (e.g.Wachsmuth et al., 1983; Vysotskaya, 1996).

We have designed 5 problems and they can be divided into two groups: first group includes problems ## 1, 4, 5 (they require a notion of conservation of density when the object is modified), while the second group consisting of problems #2 and #3 requires consideration of both parameters at the same time, along with the conservation of the relation between them.

1. 'Boat': There is a wooden boat in a pond. What will happen to the boat after it is filled with water? (consideration of density)

2. 'Raft': A raft made of 16 logs can carry 12 people. How many people can carry a raft made of 20 similar logs? (requires coherent changes in two parameters)

3.'Ships': There are 'ships' made of styrofoam cubes and nails. Each 'ship' is represented by a fraction where the nominator and denominator are equal to the number of nails and number of cubes, respectively. Order the following 'ships' (nails/cubes) from greatest to least buoyancy : 1/1, 1/2, 3/3, 2/4, 5/4, 2/3, 3/2, 2/1. (requires coherent changes in two parameters)

4. ‘Sawing a log’: A log is floating in a pond. Somebody cuts the log across resulting in two unequal pieces: one is 10 times heavier than the other. Which piece floats better2? (conservation of density)

5. ‘Joining pieces’: A small piece of wood floats in a pond. How will its buoyancy change2 if this piece of wood is attached to a side of a much bigger piece of the same wood? (conservation of density).

Problems were suggested to two groups of students (school #91, Moscow, Russia). The first group consisted of 6-graders (42 students, 10-12-olders) who did not start learning physics at school yet, so they had only their own everyday experience about the topic. The second group was the 7-graders (40 students, 11-13-olders); they had already learned the topic in their 6th grade and continued to learn the subject, but did not return to the topic again. In general, the students’ results showed that the notion of density as a ratio concept has not been assimilated by students properly.

After the assessment, we conducted a teaching experiment for 6th-graders (42 students).

5.2. The module ‘Introduction to Buoyancy’

At the beginning, weight and volume should be presented as ‘independent’ parameters. To scaffold the ‘transforming’ action at the material level, we used empty plastic bottles as ‘unit volumes’ (something that always floats) and small heavy objects (beads) as ‘unit weights’ (always sinks). We have been taping several bottles together and putting weights into them to avoid discussion about the volume of weights. We also considered the ‘suspended’ state because it is a special case where the two opposite forces are balanced. It is hard to balance a ‘vessel’ made of bottles and weights, but we are able to predict what will happen to a ‘vessel’ if we can compare it to a balanced model.

According to our hypothesis, to assimilate the ratio-concept of density properly, the students are to make up their own method of action. At the initial stage, they should be able to keep or change the buoyancy of a ‘vessel’ consciously, and later, predict the value of buoyancy if the weight and/or the volume of the object are changed.

The first set of problems supports two basic notions: (a) it is possible to change the buoyancy of an object by varying the number of weights and bottles; (b) it is possible to construct another object with the same buoyancy using different quantities of weights and bottles.

Students solved these problems using real weights and bottles and tested their ‘vessels’ by immersing them into a water tank. In each problem, students’ goal was to make a ‘vessel’ sink or float by adding or taking out some weights or bottles.

We made several attempts to organize students’ work in groups or pairs, and the best results were observed when students worked in pairs, while each of them was responsible for changing only one parameter – either weight or volume. In order to solve a buoyancy problem, the students in a pair should coordinate their actions. The joint action was a cooperative form of the ‘transforming action’ for our students.

Switching roles allows each student to learn how each parameter works and construct a correct model of the object. The model includes both weight and volume parameters, as well as the influence each of these has on the object’s buoyancy. It is the first step towards the actual abstract concept of ratio.

The transition from a ‘follow-instructions’ to a ‘design-a-procedure’ involvement level plays an important role because it immediately changes student’s learning position and attitude to lessons.

To track attempts, students locate each ‘vessel’ on a grid where vertical axis stands for weights and horizontal axis – for bottles. Each ‘vessel’ was marked as ‘floating’, ‘sinking’, or ‘balanced’. Our grid served several purposes: (a) all available vessels were thus separated into three groups: ‘floating’, ‘sinking’, and ‘balanced’; (b) it revealed the smallest balanced ‘vessel’; (c) it presented a pattern to build the entire family of ‘balanced vessels’; (d) it comprised a tool for students to plan their solutions. Also, recording the history of attempts initiates a discussion about the set of ‘vessel’ parameters chosen – thus students realize the reasons they use to distinguish between useless and useful choices. Then the history of attempts allows students to review their results and plan the next step in a more organized way.

For 6th-graders, this curriculum – exploring the effect of buoyancy and introducing the concept of density – is designed to take twelve weeks, one lesson per week.

6. Findings

We used two sets of problems to assess our course: (a) the Main Set (described above) and (b) the Modified Set of problems.

We had two control groups: the 6th graders before the instruction (they did not learn any physics course yet) and the 7th graders that have studied the topic according to ‘problem-and-practice’ curriculum in their 6th grade. The test results are shown in Table 1. The 6th-graders that have been taught ‘Introduction to Buoyancy’module took the Main Set before the instruction and the Modified Set after the instruction. The 7th-graders took both tests, with the interval between the assessments of about 3.5 months, and showed very similar results, so in what follows, we use an average of their data.

Table 01. Test results before and after instruction

Problem	6th graders before instruction (total 42 students)	7th graders (total 40 students)	6th graders experimental (i.e., after instruction) (total 42 students)
‘Boat’	26 % (11 students)	55 % (22 students)	76 % (32 students)
‘Raft’	55% (23 students)	30 % (12 students)	69 % (29 students)
‘Ships’	50 % (21 students)	30 % (12 students)	71 % (30 students)
‘Sawing a log’	40 % (17 students)	45 % (18 students)	62 % (26 students)
‘Joining pieces’	74 % (31 students)	90 % (36 students)	90 % (38 students)

Table2 shows how many students have explained their answers. The percentage is calculated only for the students who solved the problem correctly.

Table 02. Test results before and after instruction

Problem	6th graders before instruction	7th graders	6th graders experimental
‘Boat’	0	0	20 students (63% from 32)
‘Raft’	15 students (65% from 23)	8 students (67% from 12)	29 students (100%)
‘Sawing a log’	0	0	22 students (85% from 26)
‘Joining pieces’	0	0	31 students (82% from 38)

As Table 2 shows, 6th-graders after instruction gave much more explanations for solutions compared to students in control groups. Random answers prevailed in control groups; often, students have used only one word to answer, did not give any argumentation, did not want to write their name, all of which indicates random choice of answer by a student.

Low results of the 6th-graders before the instruction (problems 1 - 4) are understandable because these students have nothing to rely on in these matters but their everyday experience. In addition, all studies show that most students properly assimilate the concept of ratio only at high school.

It is more surprising to see low results shown by the 7th-graders (problems 1 - 4). Even learning the topic by doing experiments has not changed significantly students’ understanding of the density concept and their ability to apply the related skills.

The results of the 6th-graders after the instruction are considered as positive and encouraging because they are notably better than those shown by the control groups, especially for the crucial problems which required both parameters (‘Raft’, ‘Ships’).

Our results showed that the idea of conservation of density is better assimilated by the 7-graders taught according to ‘problem-and-practice’ curriculum compared to the 6th-graders before any physics instruction. However, these 7th-graders failed when dealing with problems where two parameters changed while preserving the ratio between them.

On the contrary, 6th-graders, before any physics instruction, showed better results than the 7th-graders (after teaching according to ‘problem-and-practice’ curriculum) for the problems that required keeping the value of the ratio by changing both parameters (problems ‘Raft’ and ‘Ships’). This phenomenon of non-monotonic development is well-known in many domains (e.g. Strauss 1982). In our case, we believe that the decrease in test performance can be explained by the change in the student’s strategy: they can already distinguish between two parameters, but still cannot connect them in a proper way (e.g. Siegler 2004; Siegler & Chen 2008).

7. Conclusion

The results of our study suggest that there is a special content of density as a ratio-concept that is usually left off the instruction. It may be called an ‘operational content’, whose role is to manage students’ actions while they find each parameter when the ratio is constant. The way of solving proportion problems through the discovery of the ‘compound unit’ as a special ‘construction block’ of ratio allows students to decide what they should do to find a variable and do it properly every time they face a ratio problem. Our experience shows that it takes time – more for some students and less for others – to reduce this operation to the application of a formula, i.e., to internalize this long external method to a brief mental action.

The ‘compound unit’ of the ratio used as a basement of proportional reasoning provides a possibility for every student to solve a problem through considering its particular features rather than using an abstract formula. This temporary construction works until the student deduces the formula by abbreviating the long method using a special tool provided by teaching. So, we believe that this representational system is primary in forming the abstract tier of skills within the domain.

To work with the ‘compound unit’ of ratio consciously, as opposed to ‘just one more abstract rule’, students should start from constructing it. So, all features of our module serve the goal of providing the construction of the ‘compound unit’ for students and then its internalization.

Our conclusion is that the achievement level demonstrated by our experimental group is related to the instruction. Detailed analysis shows that even performing hands-on experiments (which is a hallmark of the ‘problem-and-practice’ curriculum) does not provide students with the ratio-concept of density (working properly). We can specify three factors that we believe were the key to success of our instruction. First, by changing the students’ actions from ‘performing an experiment’ (where the goal is known only to a teacher) to ‘making an object to float or sink’ (this goal can be accepted by students), we change the students’ involvement level from ‘follow-instructions’ to ‘design-a-procedure’. Second, distributing the responsibility for manipulating parameters between students, we allow them to understand the role of each parameter, while the coordination of actions helps them to connect the variables in a proper way. Third, usage of a grid as a tool for planning further actions allows students to individually assimilate the primarily joint action.

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