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STYLE IN ARTS VS STYLE IN MATHEMATICS: ARE THERE POINTS OF CONVERGENCE?

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Abstract

The concept of style started to be systematically used in history and philosophy of science in the works of Thomas Kuhn (with his concept of 'paradigm') and Alistair Cameron Crombie in his Styles of Scientific Thinking in the European Tradition (1994). Proceeding from our interpretation of proofs as "proof-events" that take place in space and time, we approach anew the question of style in mathematics and the communicative functions of styles of proving. Proof-events are social events that generate proofs presented in different styles that describe specific mathematical practices and characterise different cultures or schools that may differ in their views of rigour. In this paper, we attempt to analyse the communicative functions of mathematical proving styles by appealing to Roman Jakobson's communication model, modified for the case of proof-events. In the framework of this model, we can discuss such questions as intelligibility of proofs, their elegance, etc. and elucidate their significance in education.

Keywords: Mathematical proof, proof-events, styles of proof, Jakobson's communication model.

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1. Introduction

In the beginning of the twentieth century, the concept of style of thinking started to be used by mathematicians (Klein, 1927; Chevalley, 1935) and physicists (Born, 1953). By the end of the same century, the concept of style of scientific thinking or other similar concepts, such as Kuhn's "paradigms" (Kuhn, 1962), Lakatos' "scientific research programs" (Lakatos, 1978), etc. started to be used also in history and philosophy of science (Mancosu, 2010). The concept of style is the cornerstone of Crombie's (1915-1996) monumental work Styles of Scientific Thinking in the European Tradition (Crombie, 1994). The style is commonly understood as a concept close to "method", "methodology", "way of reasoning" (Crombie, 1994) or as "way of writing" (Chevalley, 1935; Peckhaus, 2007). In 1999, a rich collection of papers devoted to the concept of style in mathematics was published in Moscow (Barabashev (red.), 1999).

On the other hand, the use of the term "style" in the arts is much earlier and goes back to Cicero (106 BC–43 BC) and the Greek and Roman teachers of rhetoric, who paid attention to the social and psychological effects of language. However, the concept of style was introduced and systematically used in the history of arts in Winckelmann's History of the Art of Antiquity (1764), in which the morphological difference between Greek, Greco-Roman and Roman art is determined for the first time. Thus, the concept style in the arts and philosophy of culture is generally associated with whole epochs (Winckelmann, 2006; Spengler, 1918) or with the manner of an individual author.

Have these two concepts of style any common point?

2. Research Questions

In 2014, based on the Goguen's concept of mathematical proof-event (instead of mathematical proof), in which two distinct agents (a prover and an interpreter) are involved; we suggested a definition of the style of proving as a meta-code (Stefaneas, Vandoulakis, 2014). The two agents may possess different styles of mathematical thinking or even follow different logics. Therefore, the question of communication between them arises. Although a mathematical style does not immediately generate a proof, it affects communication between the mathematicians, who may act as provers and their reader-interpreters. Thus, style may facilitate or obstruct understanding of a purported proof. There is an abundance of such cases known from the history of mathematics (Vandoulakis, 1998; 2009a; 2009b; Vandoulakis, Stefaneas, 2012; Vandoulakis, Liu (Eds), (forthcoming)).

In this paper, we attempt to analyse the communicational aspects of mathematical proving styles by appealing to Roman Jakobson's communication model. Although Jakobson's model was first formulated for literary texts, it is quite general, because it actually concerns any space of communication, which is structured in terms of six related elements: message, code, context, sender, receiver, and channel.

In the framework of this model, we can discuss such questions as intelligibility of proofs, their elegance, etc. and elucidate their significance in education.

3. Purpose of the Study

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The aim of the research is to approach from a new perspective the age-old problem of communication and understanding in mathematics.

In contrast to the traditional approach that treats mathematical activity as an exclusive product of "great minds", we set forward the communicational aspects of the mathematical activity. This can be attained by viewing proof as a social activity that takes place in time, i.e. as a proof-event, in Goguen's terminology. This implies the involvement of two agents that enact different roles – the role of prover, which can be a human or a machine or a combination of them (in the case of hybrid proving), and the role of interpreter, who generally can be a human (or group of humans) or a machine (or group of machines) or a combination of them.

The prover and the interpreter may be separated by (geographical) space and (historical) time. They may belong to different mathematical worlds, formed by their different experiences, expertise, concepts, ideas, etc. Thus, they perceive and interpret a proof differently. However, it is assumed that there is some kind of common or shared interpersonal space so that communication was made possible.

A mathematician (a human prover) may experience an insight (intention) that something in mathematics is true and communicates his experience in encoded form. The information transmitted in this way is not necessarily a proof, but a purported proof (an outline of a proof or a part of a proof, or an incomplete proof, or a conjecture). It is addressed to a potential interpreter, who will perceive and react to it by undertaking the task to decode or reconstruct or reinvent the purported proof. Thus, the stylistic shaping of a proof does not concern only the outcome of communication, i.e. the exposition of a proof (the "text"), but starts from the principles guiding the prover to select his code (or blend of codes) for the communication of his experience.

The information conveyed by a prover may generally lead to different communication outcomes: it may be characterised as plausible, or probably true, but containing gaps that have to be filled, faulty, or even may lead some interpreters to confusion (Goguen, 2001). This might be caused by many factors affecting the interaction between a prover and an interpreter.

4. Research Methods

The analysis of literary style goes back to classical rhetoric, but modern analysis of style has its roots in the school of Russian Formalism and the Prague School in the early twentieth century. Roman Jakobson, an active member of these schools, is often credited with the first coherent formulation of style, in his famous Closing Statement: Linguistics and Poetics at a conference in 1958 (Jakobson, 1960), where he exposed his theory of communicative functions of language.

Goguen & Harell (2004, 2005) have proposed a new approach to the concept of style, originating from algebraic semiotics (Goguen, 1999), which is suitable to describe styles in mathematics, because it takes into consideration structural and syntactic characteristics, as well as metaphors. They define the style in terms of the blending principles used for the construction of a blend (semiotic) space.

5. Findings

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We claim that Jakobson's model and its associated six communication functions can be used for describing the proof narratives generated in a proof-event. Prover and interpreter enact alternate roles in the course of their interaction.

6. Discussion

The referential function concerns the information conveyed in a proof-event. This information is related to the prover's use of an underlying semiotic space (the context) to describe mathematical objects or states of affairs or possibly "mental states" (intuitions, insights, intentions). Descriptive statements of the referential function can consist of terms and statements, such as "the point A lies on the circle C".

The expressive or emotive function concerns prover's disposition; it is articulated by expressions, which do not alter the denotative meaning of what is stated.

The conative function concerns the interpreter; it conveys a command to him. This is attained by sentences of the kind: "The proof is left to the reader", "The reader can easily prove", etc.

The phatic function is not related to a conveyance of information, but it concerns the channel of communication. In mathematical texts, the phatic function can be observed in phrases expressing opening, maintenance or closing of the communication channel. For instance, the word "Proof" which indicates the opening of a proof, or the symbol "■" which indicates the end of a proof.

The metalinguistic function conveys information about the code, its meaning and use. In mathematical texts, this function is observed for instance, when a prover provides explana**tions of symbols used (e.g. by** we denote the set of natural numbers), or about "idiomatic" abbreviations (e.g. the word "if" signifies "if and only if").

The poetic function is related with the (aesthetic) pleasure conveyed by mathematical discourse. This is the genuine artistic feature in mathematics. From this point of view, the information transmitted in a proof-event is viewed not only as an item of communication; the focus lies on the information conveyed per se, as well as on the code by which this information is encoded. This generates some feelings that are usually described by such terms as elegance (or, on the contrary, ugliness or clumsiness, or awkwardness) of a proof outcome (formula or a proof or a theorem). These feelings are related to what is called beauty in mathematics and many mathematicians have attempted to describe mathematics as a form of art or, at least, as a creative activity producing aesthetically assessable outcomes.

7. Conclusion

Mathematical proof is commonly considered as a universal medium of communication between mathematicians. However, proofs are only outcomes of proof-events that take place in space and time and involve particular communities. Proof-events generate proofs presented in different styles that integrate specific mathematical practices and characterise different individuals, schools or cultures. A style performs certain communicative functions that might facilitate or obstruct communication and understanding of a mathematical proof. These functions can be analysed in terms of Jakobson's communication model. The use of this model enables us to distinguish the poetic function of a proof event (as communication act), which is associated with aesthetic evaluations of a mathematical proof.

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Consequently, we may conclude that the poetic function is, in certain sense, a point of contact between mathematics and art.

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