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The European Proceedings of
Social & Behavioural Sciences
EpSBS
eISSN: 2357-1330

BE-ci 2016 : 3rd International Conference on Business and Economics, 21 - 23 September, 2016

The European Parliament after Brexit

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Abstract

http://dx.doi.org/10.15405/epsbs.2016.11.02.14

The allocation of seats in the European Parliament (EP) according to the Treaty of Lisbon, must comply with the principle of degressive proportionality. The rule can be either in the form of rounded degressive proportionality or unrounded degressive proportionality. This paper deals with expected compositions of the EP after the likely exit of the United Kingdom from the European Union bodies. We present the allocations of seats among the member states generated by means of simulation algorithms selected from the literature. These allocations are mostly unrounded degressively proportional, whereas using the approach developed in previous papers and considering all feasible allocations, we always achieve rounded degressively proportional allocations. We discuss the attained results and indicate the most appropriate allocation and methodology.

Keywords: Degressive proportionality; European Parliament; Brexit.

1. Introduction

The decision made by people voting in the United Kingdom European Union membership referendum that took place on 23 June 2016 has significantly changed many economic and political circumstances. Among other problems evidently affected by Brexit is how the composition of the European Parliament is developed. The main legal act regulating the issue is adopted in 2007 Treaty of Lisbon. Since its ratification, the binding rule of the allocation of seats among the European Union member states has been the degressive proportionality approach that departs from the classic

eISSN: 2357-1330
Selection & Peer-review under responsibility of the Conference Organization Committee

Aristotelian principle of how goods should be distributed and gives preference to smaller countries of the Community to greater countries' disadvantage. The respective provision concerning this issue reads precisely as follows: "*The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty in number, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member State shall be allocated more than ninety-six seats*" (Lisbon Treaty, p. 17).

Considering the additional specification made by the European Parliament 2007 resolution (Lamassoure & Severin, 2007), the idea of degressively proportional allocation can be expressed as a set of four conditions. Assume that $S = (s_1, s_2, ..., s_n)$ is a sequence of integer, nonnegative terms representing the numbers of allocated mandates, whereas $P = (p_1, p_2, ..., p_n)$ is a sequence of positive terms representing the numbers of populations in the respective member countries of the European Union. Then we can write that sequence S is degressively proportional with respect to P, where $0 < p_1 \le p_2 \le ... \le p_n$, if and only if

(DP 1) $s_1 \le s_2 \le ... \le s_n$,

(DP 2) $\frac{p_1}{s_1} \le \frac{p_2}{s_2} \le ... \le \frac{p_n}{s_n}$,

(DP 3) $m = s_1 = 6$, $M = s_n = 96$,

(DP 4) $H = s_1 + s_2 + ... + s_n = 751$.

Conditions (DP 1) and (DP 2) express the essence of degressive proportionality. Conditions (DP 3) and (DP 4) are arbitrarily imposed due to technical and political reasons, and we call them boundary conditions.

Mathematical analysis of the boundary conditions the reader will find in (Łyko 2012). The rule of degressive proportionality identified in this way is known in the literature as "rounded degressive proportionality" (RDP). Because of some difficulties with the consent regarding the allocation algorithm, this rule is endorsed in a weaker form as a so-called "unrounded degressive proportionality" (UDP). According to the UDP, the numbers of seats allocated are established subject to the condition (DP 2) only before rounding to integers (Ramírez González et al., 2012; Delgado-Márquez et al., 2013). A suggestion to introduce required legal amendments that would endorse the UDP, was made among others by Grimmett et al. (2011).

During the first four terms of the European Parliament, i.e. before the Treaty of Lisbon was passed, the seats were allocated in compliance with the degressive proportionality rule, even in its RDP form, although the rule as such was not explicitly referred to (Cegiełka, 2010). Afterwards, it was never the case. What is more, after the ratification of the Lisbon Treaty, the new binding rule was deliberately suspended, based on a resolution that sanctioned the case (Gualtieri & Trzaskowski, 2013). The state of affairs can be explained by at least two factors. First, the number of feasible solutions is definitely large, thus creating a space for various interpretations of the Lisbon Treaty, i.e. leading to political negotiations. For example, the sequence of population numbers in 2012 generates more than 5 million feasible allocations of seats for the term of 2014–2019. Second, and undoubtedly more important factor, is the willingness to maintain the political status quo. During all terms and in times of successive enlargements of the European Union that resulted in major changes, the great attention was paid so as not to modify much the current numbers of seats in the European Parliament allocated to respective countries. One could see it especially when the composition of the assembly for the 2014–2019 term was being formed, and the European Parliament resolution provided a stable solution defining the rule that "*nobody gains seats and nobody loses more than one*" (Gualtieri & Trzaskowski, 2013, p. 9).

As the United Kingdom withdraws the bodies of the European Community, many negative consequences may follow, however, this is also a great opportunity to sort out issues regarding the composition of the European Parliament. If we maintain unchanged (DP 3) and (DP 4), the original boundary conditions of allocation, then the number of all feasible allocations subject to the RDP for 27 countries will significantly rise. However, many of these allocations do not reduce any current number of mandates given to member countries. As a result, the most important factor disappears that impeded the adjustment of the European Parliament structure to the provisions of the Treaty. The political factor has no longer a significant influence on the distribution of seats. It is therefore worthwhile beginning again the reflections on how to specify the actual allocations that previously could not be implemented due to the mentioned political constraints. One should reconsider the recommendation stating that "the ideal alternative would be to agree on an undisputed mathematical formula of *"degressive proportionality" that would ensure a solution not only for the present revision but for future enlargements or modifications due to demographic changes"* (Lamassoure & Severin, 2007, p. 16).

2. Unrounded degressively proportional allocations

The allocation of seats in the EP inspired many algorithms that can serve as generators of its composition subject to provisions of the Lisbon Treaty. We proceed now to an analysis of a hypothetical structure of the Parliament generated by several methods that are most frequently discussed in the literature. The methods are:

- the parabolic method by Ramírez,
- base+prop (Cambridge Compromise),
- maxprop,
- base+power,
- Hamilton's generalized method

For each method, the allocations of seats in the EP will be presented in two variants: with the United Kingdom, i.e. the European Union of 28 states, and without the United Kingdom – the Community of 27 states.

The parabolic method

The parabolic method, put forward by Ramírez González (2007), is based on a quadratic allocation function $A(p) = ap^2 + bp + c$. The coefficients of the function A are chosen subject to the boundary conditions (DP 3) and (DP 4), while the i^{th} term of the sequence S equals the value of the function A rounded to the nearest integer. A detailed mathematical analysis of the allocation functions the reader will find in Słomczyński & Życzkowski (2012).

The parabolic method was one of the earliest propositions to allocate the seats in the EP. According to some Members of the Parliament, "*among the various possible mathematical formulae for implementing the principle of degressive proportionality, the 'parabolic' method is one of the most degressive*" (Gualtieri & Trzaskowski, 2013, p. 9). An additional advantage of this method is its flexibility in case of enlargements of the European Union, or after major demographic changes within the members of the Community (Moberg, 2012). The allocation by the parabolic method complies with the UDP and may not comply with the RDP, i.e. the number of seats after rounding may not satisfy the condition (DP 2).

The distribution of seats in the EP provided by the parabolic method is presented in Table 1, column I (28 states) and column J (27 states). In this case, the allocation functions are given by the formulae: $A(p) = -1,8957 \cdot 10^{-15} p^2 + 1,2668 \cdot 10^{-6} p + 5,4755$ and $A(p) = -6,8257 \cdot 10^{-15} p^2 + 1,6724 \cdot 10^{-6} p + 5,3076$, respectively.

Cambridge Compromise (base+prop)

The Cambridge Compromise (CC) is an extension of shifted proportionality proposed by Pukelsheim (2007, 2010). After further clarification of its elements made by a special symposium at Cambridge[1], it is known as a base+prop method, and the entire framework proposed at the symposium was called the Cambridge Compromise (Grimmett et al., 2011). According to this idea, each state is allocated a fixed number of seats (called the base), and the remaining seats are allocated by classical methods of proportional allocations. The participants considered different choices of base b and rounding methods when allocating the remaining seats. They eventually agreed to recommend $b = 5$ and upwards rounding to the nearest integer. In mathematical terms therefore, we deal with a linear allocation function passing through the point $(0,5)$ and subject to capping at the maximum $M = 96$[2], satisfying the condition (DP 3). The advantage of the proposed apportionment method is its simplicity and the fact that a certain number of seats are proportionally allocated. Allocations given by the CC method, as those by the parabolic method, satisfy the UDP and may not satisfy the RDP. The outcomes of the base+prop method for 28 and 27 states are presented in Table 1, columns G and H, respectively.

[1] The meeting took place in the Centre for Mathematical Sciences, Cambridge University, on 28–29 January 2011. The participants authored a report from the meeting (Grimmett et al., 2011).
[2] The first part of the condition (DP 3), m=6, is obviously satisfied given the current populations of member states.

http://dx.doi.org/10.15405/epsbs.2016.11.02.14
eISSN: 2357-1330 / Corresponding Author: Piotr Dniestrzański
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Maxprop

Allocations given by the base+prop method are relatively distant from a proportional division due to the idea of the method. The issues of discrepancy between actual allocations and proportional allocations have been studied, among others, by Karpov (2008), Łyko (2013) and Dniestrzański (2014b). As regards the Cambridge Compromise, a proportional allocation applies only to seats remaining after the base assignment of 6 mandates to each member state. An alternative for the CC can be a maxprop method proposed by Dniestrzański & Łyko (2015). The maxprop method is formulated similarly to the Cambridge Compromise. The main difference concerns the linear function applied as a basis of seat allocation, that passes through the origin, i.e. our linear function of allocation passes through the point (0,0) subject to the constraints $m = 6$ and $M = 96$, satisfying the condition DP 3. In a similar way to the CC and the parabolic method, the maxprop method satisfies the UDP and may not satisfy the RDP. The allocations generated by the maxprop method are presented in Table 1, columns E and F.

Base+power

The method of shifted proportionality and the parabolic method are often presented in official documents of the European Parliament (Lamassoure & Severin, 2007; Gualtieri & Trzaskowski, 2013) as referential for upcoming compositions of the European Parliament. It is believed that they are very close to the idea of degressive proportionality (for example Moberg, 2012). The literature also makes a case for employing a base+power method that is based on the allocation function $g(p) = b + cp^a$, ($a \in [0,1]$, $b \geq 0$, $c > 0$), where a natural number b is called a base (like in the Cambridge Compromise). The coefficient a of this allocation function may be seen as a measure of degression of an allocation generated by employing this function.

When $a = 0$ and $b = 0$, we deal with an allocation that is close to an equal division (under given rounding). When $a = 1$ and $b = 0$, we deal with an allocation that is close to a proportional division (under given rounding). In the latter case, the constraints $m = 6$ and $M = 96$ are also involved. Dniestrzański (2014a) presented one of allocations generated by means of the base+power method, employing the allocation function $g_1(p) = 5 + 174851{,}8 \cdot p^{0{,}91}$ and rounding to the nearest integer. This allocation is given in Table 1, column M. In the Eouropean Parliament without the United Kingdom, the base+power method also allows to adjust the coefficients a, b and c that yield a suitable allocation of seats. One of the options is the allocation function $g_2(p) = 5 + 149568{,}1 \cdot p^{0{,}91}$ with rounding to the nearest integer – see Table 1, column N.

eISSN: 2357-1330
Selection & Peer-review under responsibility of the Conference Organization Committee

Table 1. Theoretical allocation of seats in the European Parliament.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	P	R
Member state	Population													max	min
Malta	416,110	6	6	6	6	6	6	6	6	6	6	6	6	6	6
Luxemb-ourg	524,853	6	6	6	6	6	6	6	6	6	6	6	6	6	6
Cyprus	862,011	6	6	6	6	7	7	7	7	7	7	7	7	7	6
Estonia	1,339,662	6	6	6	6	7	7	7	8	7	8	7	8	8	6
Latvia	2,041,763	8	9	6	6	8	8	8	9	8	9	8	9	9	6
Slovenia	2,055,496	8	9	6	6	8	8	8	9	8	9	8	9	9	6
Lithuania	3,007,758	11	12	6	6	9	10	9	10	9	10	10	10	12	6
Croatia	4,398,150	11	12	6	8	11	12	11	13	11	13	11	12	13	6
Ireland	4,582,769	11	12	6	8	11	12	11	13	11	13	12	13	13	6
Finland	5,401,267	13	14	8	10	12	13	12	14	12	14	13	14	14	8
Slovakia	5,404,322	13	14	8	10	12	13	12	14	12	14	13	14	14	8
Denmark	5,580,516	13	14	8	10	12	13	12	14	13	14	13	14	14	8
Bulgaria	7,327,224	17	19	11	13	14	16	15	17	15	17	15	17	19	11
Austria	8,443,018	19	21	12	16	16	18	16	19	16	19	16	18	21	12
Sweden	9,482,855	19	21	14	17	17	19	17	21	17	21	18	20	21	14
Hungary	9,957,731	21	24	14	18	17	20	18	21	18	21	18	21	24	14
Czech R.	10,505,445	21	24	15	19	18	20	19	22	19	22	19	21	24	15
Portugal	10,541,840	21	24	15	20	18	20	19	22	19	22	19	21	24	15
Belgium	11,041,266	21	24	16	20	19	21	19	23	19	23	20	22	24	16
Greece	11,290,935	21	24	16	21	19	22	20	23	20	23	20	23	24	16
Nether-lands	16,730,348	26	29	25	31	25	29	26	31	26	31	26	30	31	25
Romania	21,355,849	32	36	32	40	31	36	32	38	32	38	31	36	40	31
Poland	38,538,447	51	58	57	73	51	60	51	60	51	60	50	58	73	50
Spain	46,196,276	54	62	69	87	60	71	60	68	60	68	59	68	87	59
Italy	60,820,764	73	84	91	96	78	92	76	82	75	82	74	86	96	74
UK	62,989,550	73	0	94	0	80	0	78	0	78	0	77	0	94	0
France	65,397,912	74	85	96	96	83	96	80	85	80	85	79	92	96	79
Germany	81,843,743	96	96	96	96	96	96	96	96	96	96	96	96	96	96
Total	508,077,880	751	751	751	751	751	751	751	751	751	751	751	751	751	751

C – 2014–2019 term; D – proportional to 2014–2019 term, downward rounding to the nearest integer; E – maxprop n=28; F – maxprop n=27; G – Cambridge Compromise n=28; H – Cambridge Compromise n=27; I – parabolic n=28, $A(p) = -1,8957 \cdot 10^{-15} p^2 + 1,2668 \cdot 10^{-6} p + 5,4755$; J – parabolic n=27, $A(p) = -6,8257 \cdot 10^{-15} p^2 + 1,6724 \cdot 10^{-6} p + 5,3076$; K – generalized Hamilton n=28, $A(p) = -2,0293 \cdot 10^{-15} p^2 + 1,2778 \cdot 10^{-6} p + 5,4709$; L – generalized Hamilton n=27, $A(p) = -6,9601 \cdot 10^{-15} p^2 + 1,6834 \cdot 10^{-6} p + 5,303$; M – base+power, $g_1(p) = 5 + 174851,8 \cdot p^{0,91}$, rounding to the nearest integer, n=28; N – base+power, $g_2(p) = 5 + 149568,1 \cdot p^{0,91}$, rounding to the nearest integer, n=27; P – maximum values among allocations for n=27; R – minimum values among allocations for n=27.

http://dx.doi.org/10.15405/epsbs.2016.11.02.14
eISSN: 2357-1330 / Corresponding Author: Piotr Dniestrzański
Selection and peer-review under responsibility of the Organizing Committee of the conference

Hamilton's generalized method

This method was proposed by Cegiełka & Łyko (2014) who combined classical proportional allocations with degressive proportionality. First, a real sequence of values of the quadratic allocation function is determined $A(p)=ap^2+bp+c$, subject to the boundary conditions (DP 3) and (DP 4), i.e. $A(p_1)=m$, $A(p_n)=M$ and $\sum_{i=1}^{n} A\left(p_i\right)=H$. Next, each i^{th} state is assigned a number of seats equal $\lfloor A(p_i) \rfloor$, i.e. an integer part of the value of function A. The remaining seats are allocated to states with largest fractional part, one mandate to one state. Hence, the generalized Hamilton's method employs the classical Hamilton's method (also known as the method of largest remainders) in the area of degressively proportional allocations by indicating a sequence of real terms (regarded as an analogy of a quota sequence in proportional allocations) and rounding the generated numbers by Hamilton's method (for the proportional methods of apportionment see for instance Balinski & Young, 1982, Pukelsheim, 2014). Hamilton's method satisfies the UDP and may not satisfy the RDP. The allocation of seats generated by Hamilton's method is presented in Table 1, columns K and L. The allocation functions employed to determine the composition of the EP are given by the formulae:

$A(p)=-2{,}0293\cdot 10^{-15}p^2+1{,}2778\cdot 10^{-6}p+5{,}4709$ and

$A(p)=-6{,}9601\cdot 10^{-15}p^2+1{,}6834\cdot 10^{-6}p+5{,}303$, for 28 and 27 states, respectively.

Table 1 shows allocations of the EP seats generated by all above reviewed method and the allocation that is proportional to the current composition (2014–2019 term), with downward rounding to the nearest integer.

Analysis

Allocations presented in Table 1 demonstrate the influence of the choice of algorithm on the composition of the Parliament. For example, Spain can be assigned a number of seats from 68 to 87, depending on the underlying criterion. The difference between the most favourable and the least favourable allocation is 19 seats, or about 28 percent. In case of Croatia and Ireland, the differences between the maximum and the minimum are even larger, more than 62.5 percent, or 5 seats in absolute terms. Clearly, the discrepancies in case of smallest states are relatively insignificant (Luxembourg, Estonia), because the boundary conditions effectively prevent the allocation of seats significantly larger than the minimum $m=6$. A particularly exceptional situation is that of Lithuania. The maximum number of seats allocated to this state is larger from the minimum by about 67 percent, however, what is most peculiar, any method of allocation (except for the allocation proportional to the current one; Table 1, column D) assigns to Lithuania less seats by at least one. The prospective absence of the United Kingdom in the Community will result in takeover of British 73 seats by remaining member states. One could certainly expect that all those remaining members of the Community should "gain" after Brexit, or at the very least, not to lose. In case of Lithuania, all the algorithms take their seats from them. Besides, there are states that do not gain anything (except for the allocation proportional to the current one, Table 1, column D), i.e. Malta, Germany, Luxembourg, Bulgaria, Austria and Hungary. In case of Malta and Germany, it is an obvious consequence of the boundary condition (DP 3), but the other four can hardly agree to such a result.

In addition to the number of allocated seats, an equally important issue when deciding about the composition of the Parliament is the compliance with the binding law. The report by Lamassoure & Severin (2007) states that a fundamental component of the degressive proportionality principle included in the Treaty of Lisbon is the condition (DP 2) dealing with rounded degressive proportionality. All above analyzed methods may be incompatible with the RDP. A detailed analysis of allocations in Table 1 demonstrates that, in fact, such incompatibility occurs quite frequently. The entries in Table 1 printed in boldface signify cases when the condition (DP 2) is not satisfied. The allocations of seats in the Parliament without the United Kingdom satisfy the RDP only in case of the parabolic method and the generalized Hamilton's method. The outcomes are not stable and depend on demographic shifts, accessions (or more exits) resulting in the failure to comply with law.

3. Rounded degressively proportional allocations

The inability of algorithms described in section 2 (or based on other functions of allocation) to always return a rounded degressively proportional allocation motivates researchers to find other methods leading to degressively proportional allocations. One of the results is the algorithm LaRSA presented by Łyko and Rudek (2013) that determines a set Π of all feasible RDP allocations subject to specified boundary conditions. Given the current required values of m, M and H and 27 Member States, the algorithm terminates after a relatively short time (about 6 hours). It turns out that the prospective exit of the United Kingdom from the bodies of the European Union significantly influences the number of all allocations. With 28 states, the number of all feasible solutions was over 5 million, whereas the reduction to 27 members of the Community increases the number of solutions to almost 6 billion. In addition, what is important in political terms, there are 3.6 billion solutions with no losses of mandates allocated to each state. Thus, a new, yet unexplored space becomes open for negotiations. One of the main obstacles to reaching an agreement was always the inevitability of reducing the number of seats allocated to some countries.

The aforementioned algorithm LaRSA also returns a minimum $s_i^{\min}$ and a maximum $s_i^{\max}$ number of mandates assigned to each country in an RDP allocation. The respective values are presented in Table 2, columns E and F. Considering the differences between the entries in columns E and F, and also the ratios of these differences to respective values of $s_i^{\min}$, we determine a range of absolute and relative variations of feasible seat numbers under rounded degressively proportional allocation. The numbers of seats for Malta and Germany are determined by the condition (DP 3), so they do not vary, hence we exclude the two countries from this analysis. The largest absolute difference of 29 seats occurs in case of Poland. The largest relative extent of more than 230 percent relates to Latvia and Slovenia. It must be underlined that the maximum number of mandates assigned to Italy is 96 and is constrained by the boundary condition. Disregarding this, the smallest relative variation of almost 50 percent is found in case of Spain.

http://dx.doi.org/10.15405/epsbs.2016.11.02.14
eISSN: 2357-1330 / Corresponding Author: Piotr Dniestrzański
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When comparing the numbers $s_i^{\max}$ and $s_i^{\min}$ with the numbers of currently held mandates shown in Table 2, column D, we derive several interesting conclusions. First of all, the maximum feasible number of mandates that may be assigned to each country is greater than the numbers of currently held seats in the European Parliament. In this perspective, the largest gain of 28 seats following the possible changes may be realized by Spain. In relative terms, the most favourable changes may affect Estonia whose gain of 11 mandates means an increase by more than 180 percent versus currently held 6 mandates.

France and Spain are similarly positioned as their minimum possible allocation of seats equal 77 and 55, respectively, is greater than 74 and 54 mandates held at present. These countries will also gain when the forthcoming allocation is rounded degressively proportional. There are also five countries comprising Luxembourg, Cyprus, Estonia, Croatia and Ireland that never lose compared to their current allocations, regardless of the specific RDP allocation. At present, the seven mentioned countries suffer mostly a disadvantage of degressive allocation in the current term. Oddly enough, the countries in this group are small, medium and large as regards their populations. This fact may imply that the current allocation of seats in the European Parliament is quite random.

The unfavourable changes may affect mostly Romania, with possible loss of 6 mandates in the worst case. In relative terms, Lithuania may suffer from the most unfavourable change, with loss of 3 mandates, or more than 27 percent of what they currently hold. Nevertheless, due to expected redistribution of 73 mandates assigned currently to the United Kingdom, a new allocation globally will result in more gains than losses. The total number of seats to be lost by member states is 49, while the total number of seats to gain, at the maximum, is 291.

Analyses presented in section 2 (Table 1, columns F, H, J, L and N) prove that no allocation method ensures that no country will lose compared to the current allocation after the exit of the United Kingdom. This seems a major weakness of these methods. With regard to classical thinking about proportional division, we even deal with a unique paradox. In spite of less states competing for seats, there is one country suffering a loss at their currently held seats. Thus, another method of allocation is needed that will allow to maintain the political status quo.

One of the solutions that is free from the mentioned weakness is presented by Łyko & Rudek (2013). The authors employ the allocation function defined by the formula $A(t) = M\frac{t-p_1}{p_n-p_1} + m\frac{p_n-t}{p_n-p_1}$. The function transforms proportionally a segment $[p_1, p_n]$ to a segment $[m, M]$. As a result, one may say that a sequence of populations is proportionally mapped to another interval containing terms of a sequence that represents a whole number allocation. The cited paper proposes to find a required allocation as a sequence of integer terms $S = (s_1, s_2, ..., s_n)$ satisfying the conditions (DP 1)–(DP 4) and minimizing a sum of squares of distances from the sequence $A(P)$, i.e. $\sum_{i=1}^{n}\left(A(p_i)-s_i\right)^2$, over the set Π. Accordingly, in a sense of a chosen metric, this sequence also satisfies one of additional postulates known in the literature related to degressively proportional allocation of seats in the European Parliament, namely that "*the minimum and maximum numbers set by the Treaty must be fully utilised to ensure that the allocation of seats in the European Parliament reflects as closely as possible the range of populations of the Member States*" (Lamassoure & Severin, 2007, p. 8). The allocation determined in this way is presented in Table 2, column M.

Table 2. Constraints to seat numbers in the European Parliament, compatible with RDP, and proposed composition of the EP without the UK.

A	B	C	D	E	F	G	H	I	J	K	L	M
	Member state	Population	Current									
1	Malta	416,110	6	6	6	0	0%	0	0	0%	0%	6
2	Luxembourg	524,853	6	6	7	1	17%	0	1	0%	17%	7
3	Cyprus	862,011	6	6	11	5	83%	0	5	0%	83%	11
4	Estonia	1,339,662	6	6	17	11	183%	0	11	0%	183%	12
5	Latvia	2,041,763	8	6	20	14	233%	-2	12	-25%	150%	12
6	Slovenia	2,055,496	8	6	20	14	233%	-2	12	-25%	150%	12
7	Lithuania	3,007,758	11	8	21	13	163%	-3	10	-27%	91%	13
8	Croatia	4,398,150	11	11	22	11	100%	0	11	0%	100%	15
9	Ireland	4,582,769	11	11	22	11	100%	0	11	0%	100%	15
10	Finland	5,401,267	13	12	23	11	92%	-1	10	-8%	77%	16
11	Slovakia	5,404,322	13	12	23	11	92%	-1	10	-8%	77%	16
12	Denmark	5,580,516	13	12	23	11	92%	-1	10	-8%	77%	16
13	Bulgaria	7,327,224	17	15	25	10	67%	-2	8	-12%	47%	18
14	Austria	8,443,018	18	17	26	9	53%	-1	8	-6%	44%	20
15	Sweden	9,482,855	20	17	28	11	65%	-3	8	-15%	40%	21
16	Hungary	9,957,731	21	17	28	11	65%	-4	7	-19%	33%	21
17	Czech Rep.	10,505,445	21	17	28	11	65%	-4	7	-19%	33%	22
18	Portugal	10,541,840	21	17	28	11	65%	-4	7	-19%	33%	22
19	Belgium	11,041,266	21	17	29	12	71%	-4	8	-19%	38%	22
20	Greece	11,290,935	21	17	29	12	71%	-4	8	-19%	38%	22
21	Netherlands	16,730,348	26	21	40	19	90%	-5	14	-19%	54%	29
22	Romania	21,355,849	32	26	48	22	85%	-6	16	-19%	50%	34
23	Poland	38,538,447	51	46	75	29	63%	-5	24	-10%	47%	53
24	Spain	46,196,276	54	55	82	27	49%	1	28	2%	52%	61
25	Italy	60,820,764	73	72	96	24	33%	-1	23	-1%	32%	77
26	UK	62,989,550	73					-73	-73	-100%	-100%	
27	France	65,397,912	74	77	96	9	25%	3	22	4%	30%	82
28	Germany	81,843,743	96	96	96	0	0%	0	0	0%	0%	96

D – 2014–2019 term; E – $s_i^{\min}$; F – $s_i^{\max}$; G – $s_i^{\max}-s_i^{\min}$; H – $\left(s_i^{\max}-s_i^{\min}\right)/s_i^{\min}$; I – $s_i^{\min}-\text{current}$; J – $s_i^{\max}-\text{current}$; K – $\left(s_i^{\min}-\text{current}\right)/\text{current}$; L – $\left(s_i^{\max}-\text{current}\right)/\text{current}$; M – proposed RDP allocation.

It is certainly rounded degressively proportional. Thus, there is no need to relax the requirements concerning the levels of UDP sequences, and the solution obtained better reflects the original, legally declared idea of degressive proportionality. In addition, no member country is assigned less mandates than currently, with Hungary being a single country assigned the same number of seats they are holding now. Therefore, the scheme is politically neutral, does not reduce the number of representatives of any country and there is no need to withdraw a mandate. It is also worth underlining that compared to proportionally increasing numbers of seats in the European Parliament shown in Table 1, column D, the proposed RDP allocation is more advantageous for less populated states, i.e. all countries with populations smaller than that of Bulgaria. These states, except for Malta, gain at least one mandate, with Estonia gaining at the utmost, i.e. 6 mandates more than in case of proportionally incrementing seat numbers. Other states, with populations larger or equal to that of Bulgaria, get numbers of seats less or equal to those resulting from proportionally incrementing seat numbers. The largest difference is recorded by Italy, with 77 seats after the RDP allocation, and 84 seats after proportional increments. The proposed RDP allocation correctly reflects the variances of populations in member states and also better implements the motto of the European Union "*United in diversity*" (Official Journal of the European Union, p. 357).

4. Conclusion

The exit of the United Kingdom from the European Union results in a significant increase of number of RDP allocations. More than half of them are advantageous for all the member states in the sense that each state gets at least the same number of seats as currently held. Undoubtedly, 3.6 billion options offer a wide bartering area to bodies that decide upon the composition of the Parliament. On the other hand, reviewing all feasible solutions from such a huge pool is practically unworkable. The choice of a criterion that will actually indicate one specific solution definitely improves the entire process of allocating seats. Our proposition of a minimizing criterion has two significant strengths. On the one hand, it reflects the variance of population in the member states, while on the other hand, it ensures a suitable representation of smaller states. As a result, it combines the main ideas of the composition of the EP. The algorithm maybe is not a simple method, but it generates all existing RDP solutions. The criterion itself can be modified according to a sense of fairness or in the course of political negotiations. As a result, regardless of where the emphasis is put with regard to the composition of the EP, an RDP allocation will be always ensured. The exit of the United Kingdom from the Community results in numerous allocations, and many of them satisfy the postulate of maintaining the numbers of mandates at least at the same level, therefore, it seems inappropriate to relax the degressive proportionality requirements and to consider the UDP allocations.

Acknowledgements

The results presented in this paper have been supported by the Polish National Science Centre under grant no. 2013/09/B/HS4/02702.

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