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Learning Abilities and Geometry Achievements

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Abstract

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Formal geometry is the most problematic subject in middle school and high school math studies. The difficulty stems from the need to understand the language of mathematics in the field of geometry while integrating it with prior knowledge, parallel to the level of the student's mental development.

To help students overcome the difficulties in learning geometry a program was developed based on a tool combining use of teaching strategies, through mediation, such as proper use of mathematical and geometric language in particular, including visual marking and identifying memory supports to prior knowledge and using it in the process of finding a solution

This presentation seeks to present findings emerging from research that examined the effect of use of a Learning Strategies Program that combines teaching strategies in solving geometry problems on 10th Grade pupils' achievements, their attitudes towards learning Mathematics in general and Geometry in particular.

A Geometry level Van Hiele questionnaire (as developed by Patkin, 1990) and Raven test (2000) mapped the learning abilities of the pupils. The pupils' knowledge regarding solving geometry problems was tested.

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Keywords: Geometry, Strategies, Pupil's attitudes, Learning Strategies Program (LSP), Mediation tool.

1. Introduction

Geometry is one of the most important branches of mathematics education, because the aim of the geometry teaching is to provide pupils with the ability of critical thinking, problem solving, and a better understanding of the other subjects in mathematics by making the pupils have a high level of geometric thinking skills (Şahin, 2008).

Difficulties arise from the need to understand mathematical language in the field of geometry, while integrating it with prior knowledge, in parallel to the pupils' level of mental development.

Teaching geometry is a difficult task for teachers. The difficulties require excellent teachers with adequate training and abilities so that they can carry out the task of mediation between material and learner.

To help pupils overcome learning difficulties in geometry, a program was devised that combines use of teaching strategies.

This program integrates focused teaching strategies acquired by pupils through classroom teaching that give pupils tools to cope with solving other problems. Using this strategy is appropriate for every pupil, does not require repetition or oral memorization by pupils, which have not proven to be tools for solving problems in geometry. Using these strategies enable pupils to develop skills to solve problems at any level. Teaching these strategies will be implemented through mediated teaching in class by teachers in parallel with teaching the contents of the curriculum.

2. Theoretical Framework

Geometric shapes in general and polygons in particular comprise one of the main topics that accompany pupils from elementary to junior high school. Geometry is perceived by many pupils as the study of formal proof, boring and not understood, and this is because teachers devote lots of time to learning proof, and do not devote any time to developing other important skills, on which geometry is based (Hoffer, 1981).

The important skills for studying geometry according to Hoffer (1981) are: **visual skills, verbal skills, sketching skills, scoring, logic skills, practical skills**. According to Hoffer a combination of all the aforementioned skills in teaching geometry, will contribute to increasing pupils' interest in the subject and their understanding of learned material. Acquiring these skills is the basis for learning geometry, however this is not enough. There is also a need for children to reach a required level of mental development in order to understand geometry.

Van Hiele's Theory (1959) is the main theory engaging in the development of geometrical thinking. The theory discusses the various stages of mental development of geometry learners. Acquisition of skills is fundamental to the learning of geometry, but it is necessary for pupils to reach a level of mental development required for understanding geometry. Geometry learners' mental development can be hierarchically arranged on five levels. Sarfaty and Patkin (2011) listed the levels of geometric thinking: **Recognition, Analysis, Ordering, Deduction, Rigor**.

Piaget (1960) and other researchers have emphasized the covert cognitive processes occurring within the learner, meaning the learner's individual development. In contrast, Vygotsky (1978) emphasized social-cultural processes as the source of intrinsic cognitive change. Vygotsky argued that developmental processes and learning processes are not the same. The developmental process follows learning. The latter takes place in the zone of proximal development and the learning process becomes a developmental process (Miller, 2011). According to Vygotsky, what exists within the child as Zone of Proximal Development will appear in the future as actual development.

A strategy of teaching in stages, according to Galperin (1992a, 1992b), serves as a guideline for teachers and not as a collection in a teaching guide that must accompany every individual. He particularly emphasized four unique stages (steps) of the teaching-learning process: (1) orientation, (2) communicated thinking, (3) dialogical thinking and (4) acting mentally. The nucleus of teaching and learning is found in conceptualizing mental process and abilities originating in significant shared activities, presenting children with such activities, giving them cognitive tools and directions to a process that leads to their development. These ideas can help develop a framework in which there is a more profound reference to mutual influences between teaching, learning and development.

Teaching geometry is a difficult task for teachers. Geometry teachers need to develop expertise in the formal part of the subject and its combination with verbal explanations, which have special wording geometry. These difficulties require teachers with high discourse capacity, who are qualified and capable so that they can fulfill the task of mediating the material to the learner in a heterogenous group of learners while using varied tools and skills and developing solution strategies that will constitute a significant and meaningful mediation tool.

Teaching strategies are defined as ways that the teacher takes to accomplish the objectives of the lesson (Schroeder, Scott, Tolson, Huang & Lee, 2007). Learning strategies are a way to improve teaching and learning. In addition, they are a series of cognitive processes that influence information processing in order to provide pupils with tools that will help them learn, solve problems, and complete tasks independently. Strategies of teaching formal geometry are based on the characteristics of the processes which allow for quality teaching-learning-assessment processes as defined in Director General Circulars (2012) in Israel.

Mathematical discourse requires a process of mediating the causality of mathematical phenomena between teacher and pupil (Regev & Shimoni, 2000). Therefore, the discourse is guided by presenting a problem and finding possible ways of solution.

The proposed program is based on Feuerstein's approach (1998). Feuerstein believed the pupil is able to acquire not only knowledge and skills but also new cognitive structures and this realization requires an investment of effort and resources. This ability is imparted to pupils through mediated learning in which he or she has direct interaction with the environment through direct exposure. This direct interaction and exposure provides the learner not only knowledge or skill, but also ways of observation, approaches and ways of finding the link between them.

3. Research Justification

In order to help pupils overcome difficulties in learning geometry, a program was developed, which integrates teaching strategies such as correct use of the geometry language, with visual scoring and finding memory supports for prior knowledge and using it to solving problems in geometry.

This research seeks to examine the influence of Learning Strategies Program (LSP) intervention program on pupils' achievements in geometry and their attitudes towards mathematics and geometry. The article relates to the issue of the contribution of a strategy based program to pupils' achievements in and their attitudes towards geometry and mathematics.

4. Previous Studies on Similar Programs

Different studies (Hershkovitz, 1992; Kramersky, 1996; Pelach-Borowitz, 2004; Shalev, 2002) have dealt with the link between using teaching strategies and pupils' achievements. Practice strategies based on computers, using drawings in geometry and Van Hiele's levels of thinking model (Idris, 2009; Kutluca, 2013; Patkin, 1994) to solve problems in geometry as well as different strategies used by mathematics teachers in teaching computational geometry, trigonometry (Aydoğdu, 2014). There are those that dealt with different factors and thinks between them and the ability to solve problems in geometry, such as: motivation, feelings, drawing skills (Bailey, Taasobsshirazi & Carr 2014).

No studies were found that dealt with the construction of models of strategies in teaching geometry which constitutes a mediation discourse between teacher and pupil and their use in high school.

As such, the researcher chose to conduct research whose aim is to examine the influence of a mediated strategic tool in geometry and its use on pupils' achievements in solving problems in geometry and their views with regard to mathematics and geometry in particular.

5. LSP Program

The Learning Strategies Program (LSP) includes six sessions of approximately 4 hours per session. Each session includes a teaching outline of the tool and employing it while teaching the deductive language of geometry, and at the same time, teaching the "Thinking Persons" tool and instructions of how to use it.

5.1 The Strategic Tool - 'Thinking Person'

Visualization - The "Thinking Person" is a permanent symbol that constitutes a mediating tool between pupil and teacher during geometry lessons. The tool includes strategies for solving geometric problems and thinking. The tool is a mediating discourse tool between teacher and pupil and represents for them both a lots of information.

The smiley face symbol is easy to draw and identify. The symbol is a smiling face with a pair of horns, each of which as a meaning as a mediating dialogue tool - as thinking channels. One channel, which contains arches that demonstrate continuous thinking. The second channel characterizes the connection between conclusions derived from the arches in the first channel that represent data. This channel represents the stage of reaching widening conclusions - what is possible to be done with problem givens and the conclusions derived from these givens.

Linguistic meaning - in the first channel each arch is characterized by a geometric language regarding the logic of 'if 'then'. This channel serves as an memory support to the continuing thinking process that pupils have to go through when in the thinking process while solving a problem (Hoffer, 1981) and according to progress in required levels of thinking through exercises presented to pupils (Van Hiele, 1959).

When pupils build an arch in the first channel, they brainstorm in order to understand given meanings that appear in a problem. When they know that they have to create thinking arches, they internalize that they must carryout of process of thinking about prior knowledge. The givens allow

them to focus on specific prior knowledge and they brainstorm about knowledge linked to this given, and bring them forward into their immediate memory, which allows them to draw from memory on prior knowledge in a way that will not require repetition or remembering sentence by heart.

Pupils use the second channel of thinking people when they have to gather all the givens in a problem. After they have gone through all stage of building all possible arches using the first channel. In the second channel, pupils mix the information in order to draw conclusions that they are permanent processes: overlapping triangles (all sentences), and then it is possible to add various advance processes using the curriculum such as: triangles (all sentences), trigonometry - sines and cosines and more, according to the curriculum.

During the process of using the thinking person, it is only afterwards that it becomes an integral part of cognitive structures belonging to pupils (Galperin, in Arieivitch & Haenen, 2005). When teachers instill a taught subject, they must present the thinking person not just visually. They must explain to pupils the meaning of the channels and demonstrate to them while solving a problem, the solution process according to the channels and using this tool.

The proposed program uses learning strategies that will lead the pupils, through experiencing the process, to personal, informed understanding in the interaction with adults and the community, as well as emphasizing the pupil's uniqueness. The goals will be accomplished as a result of the pupil's positive experience in the mediating discourse, which will be discussed later on.

The process of finding a solution by the teacher and the pupil together, with correct use of geometric language while using visual scoring, finding memory supports for prior knowledge and its use, including asking questions, will advance the pupils in authentic mathematical activity, encounter them with problems, methods and many solutions, advance their creativity and perhaps even serve as a bridge for pupils' thinking from concrete to abstract both in geometry and coping with abstract situations in life.

5.2. Aims of the LSP

The goal of this Learning Strategies Program (LSP) is to give the pupil's and teacher a meaningful mediation between material and groups of heterogeneous learners to generate complete and quality mediated learning.

This program integrates focused teaching strategies which the children acquire in class. These strategies provide the pupils with the tools to cope with solving other problems. Use of the strategy suits all pupils, does not require memorization, which has not proven effective as a tool for solving geometric problems. It allows the pupils to develop problem solving skills at any level. Teaching of the strategies will be implemented by the teacher in parallel to teaching the contents.

5.3. Unique Characteristics of the LSP

For the teacher to produce whole quality mediated learning, the Learning Strategies Program uses the tools of dialogic discourse. Mathematical discourse requires a process of mediating the causality of mathematical phenomena between teacher and pupil (Regev & shimoni, 2000).

The innovation in integrating this mediated strategic tool is that it provides a holistic response to pupils', as well as teachers', needs in teaching geometry using one common tool. This tool provides a

response to teachers' as well as pupils difficulties and through it both sides are given a solution in the learning process as well as a solution to problems in geometry.

6. Methodology

In order to check the research question and hypothesis a quantitative research approach and tools were chosen.

6.1 Research Population

The research participants are 15-16 years old pupils in three 10 grade classes, in a high school in central Israel. Pupils learn mathematics according to the curriculum determined by the Ministry of Education at a four point level¹. Thus, the experimental design will include two groups:

Experimental group – one class that studied the subject according to the unique program integrating strategies for solving problems in geometry- Group 3 (N=24)

Control group - two classes, the same age group that studied the issue in accordance with the regular curriculum without the integration of unique teaching strategies - Group 1 (N=26) and Group 2 (N=27).

6.2 Research Aims

1. To investigate the contribution of Learning Strategies Program on pupil's achievement in Geometry.
2. To examine the effects of using a Learning Strategies Program in Geometry on the pupil's problem solving skills.
3. To explore the differences between the experimental group and the control groups in attitudes towards mathematics in general and geometry in particular.

6.3 Research Questions

1. In what way has the Learning Strategies Program influence the pupils' achievements in geometry?
2. What is the contribution of the Learning Strategies Program to the pupils' ability to solve problems in geometry?
3. Are there differences between the experimental group and the control groups regard to attitudes to mathematics in general and geometry in particular?

6.4 Research Hypotheses

1. The experimental group will have higher results in a geometry test at the end of the program.
2. The experimental group will demonstrate greater improvement between the first and second test.

¹ Israeli high school subjects are taught on the basis of points, or credits – the highest level is 5 points; 4 points is the second highest level etc.

3. Analysis of pupils' means of solving problems will show a difference between the groups with regard to problem solving skills.
4. The experimental group will demonstrate more positive attitudes towards mathematics and geometry.

6.5 Research Tools

A. Research Tools for Examining the Initial Level in the Three Groups

1. Testing previous knowledge and understanding in geometry, polygons, triangles, parallelisms (Patkin, 1990) - To test pupils' knowledge and understanding.
2. Raven test (Raven,2000) - To examine general non-verbal cognitive levels, and evaluate development of thinking processes from the simplest level to analogous thinking.
3. Van Hiele Questionnaire (as developed by Patkin, 1990) - To examine different levels of thinking in geometry in accordance with Van Hiele's theory. This test was administered in the beginning to assess initial level, but also at the end of the program.

B. Research Tools for Examining Achievements in Geometry

Geometry achievements tests (Test 1, Test 2) - To examine pupils' achievements during the interaction and at its end with emphasis on level of achievement and use of mediating tool.

C. Research tools for Examining beliefs about Mathematics

We will employ Pinchevsky's (2001) tool, the purpose of the questionnaire is to examine beliefs regarding mathematics according to three measurements: belief regarding the nature of mathematics, beliefs regarding means of teaching and learning resolving mathematical problems and beliefs regarding self-efficacy in mathematics learning. The questionnaire will be distributed to participants at the beginning and end of the intervention program.

D. Research Tool for Examining Attitudes regarding Geometry

The questionnaire was developed by Patkin (1990), its items refer to a number of issues in the cognitive and effective areas, where pupils are asked to express their opinion on the subject of geometry and its importance, on teaching methods, on achievements and the relationship of pupils in general and each individually to the subject. The questionnaire will be distributed to participants at the beginning and end of the intervention program. The questionnaire's reliability is Cronbach's $\alpha = 0.84$.

7. Findings

The findings chapter will be divided into four parts. The first part findings from an examination of the starting levels of the three research groups will be presented. The second part will present findings that examined the difference in geometric achievements. The third part will present finding that examined pupils' attitudes to geometry and the fourth part will present findings that examined pupils' beliefs regarding mathematics.

Knowledge and Understanding of Geometry before the LPS intervention

Table 1. Variance results of comparison of groups in Geometry level test

	Group 1 N=26		Group 2 N=27		Group 3 N=24		total	group		
	mean	Std. deviation	mean	Std. deviation	mean	Std. deviation		mean	Std. deviation	F
Knowledge	10.077	3.7728	11.444	2.2927	9.917	3.4125	10.506	3.2388	1.871	0.161
Comprehension	15.038	2.6904	16.370	2.1865	15.458	4.0323	15.636	3.0432	1.358	0.264
Application	10.077	1.9985	9.185	1.8405	8.375	3.4365	9.234	2.5541	2.900	0.061
Analysis & synthesis	13.000	6.1838	17.667	4.7150	16.250	4.9454	15.649	5.6166	5.191	0.008
Total	48.192	10.5528	54.667	6.1080	50.000	13.6286	51.026	10.6487	2.821	0.066

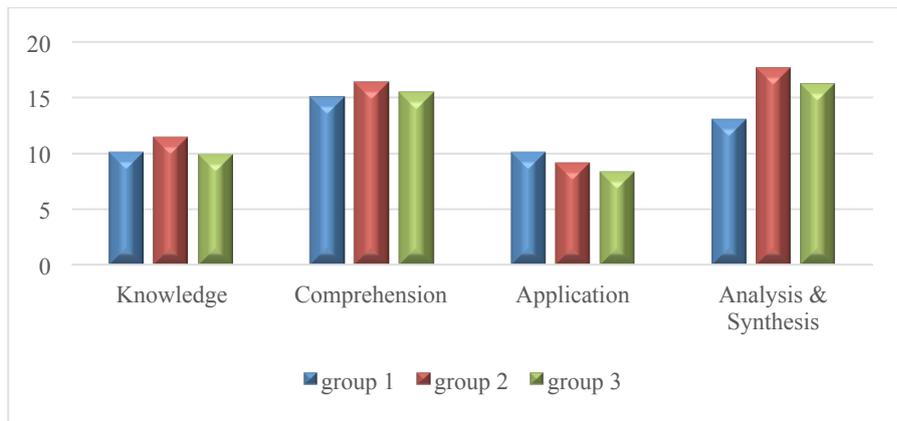


Figure 1. Geometry level test - comparison of groups

Data analysis was carried out according to van Hiele's levels of thinking. It was found that in three of the four criteria, control group 2 received the highest scores in relation to the other two groups. Differential analysis was carried out on each of the variables, the results showed that there is no significant difference in the achievements at each level (Knowledge, Comprehension, Application), with the exception of Analysis & Synthesis, where a significant difference was found in the level of thinking [F(2.73)=5.191; p=0.0080] between the groups. Bonferroni post hoc test found that group 1 differed from group 2 [M=17.667 (Sd.= 4.7150), M= 13.000 (Sd.= 6.1838) respectively].

Table 2. Results of Raven test for comparison between groups

Group 1 N=26		Group 2 N=27		Group 3 N=24		total			
mean	Std. deviation	mean	Std. deviation	mean	Std. deviation	mean	Std. deviation	F	Sig.
71.000	20.88444	76.3704	20.04063	65.8333	23.46258	71.2727	21.58864	1.689	0.192

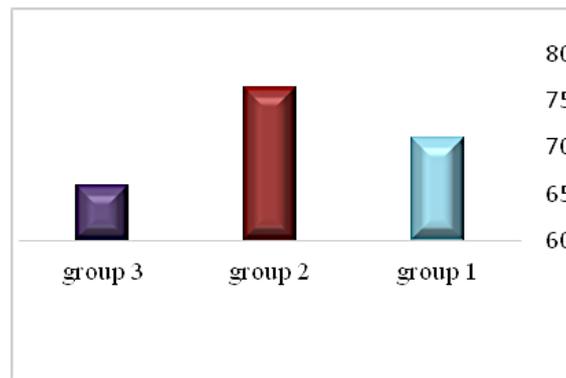


Figure 2. Raven test – comparison of groups

In order to examine differences between the three groups, a one-way ANOVA analysis was carried out between the groups. It was found that there were no significant differences between the groups, however it is possible to say that control group 2 got the highest score ($M=76.370$ ($Sd.=20.040$)) in the test and experimental group 3 got the lowest score ($M=65.833$ ($Sd.=23.462$)).

Table 3. Results of examination of differences between groups before and after their interaction – Van Hiele test

	Group 1 N=26		Group 2 N=27		Group 3 N=24		total		F	Sig.
	mean	Std. deviation	mean	Std. deviation	mean	Std. deviation	mean	Std. deviation		
Pre-van	3.5385	2.61122	5.6296	4.27108	5.4167	3.83255	4.8571	3.71949		
Post-van	5.5769	3.95144	6.3333	2.80110	6.8750	5.05889	6.2468	3.98073		
time									3.308	0.073
Time*group									0.548	0.580
group									2.033	0.138

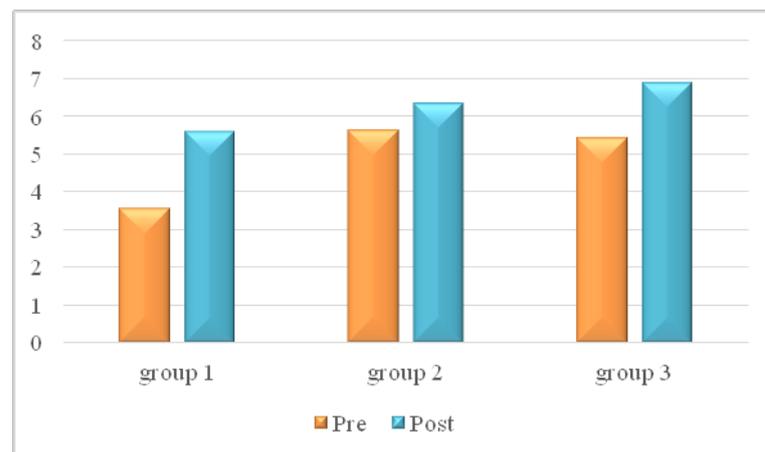


Figure 3. Van Heile test before and after the interaction - Comparison of groups

In order to examine differences between the three groups before and after and the interaction between time and group, a two-way ANOVA analysis was carried out between the groups. It was found that there were no significant differences between the groups [$F(2,73)=2.033$; $p=0.138$]. The time-group interaction did not yield significance either [$F(2,73)=0.548$; $p=0.580$].

It was found there were no differences between the groups before and after the interaction. However, improvements were seen in the mean scores in the experimental group 3.

7.1 Differences in achievements in geometry during and after the LSP

Table 4. Results of differences between groups and between times during and after the interaction in a Geometry achievements tests (Test 1 during and test 2 after the interaction)

	Group 1 N=26		Group 2 N=27		Group 3 N=24		total		F	Sig.
	mean	Std. deviation	mean	Std. deviation	mean	Std. deviation	mean	Std. deviation		
Test 1	56.923	37.9230	75.556	35.4459	72.083	30.9248	68.182	35.4938	2.033	0.138
Test 2	66.346	34.1631	77.778	34.5669	84.167	23.7591	75.909	31.8996	1.829	0.168

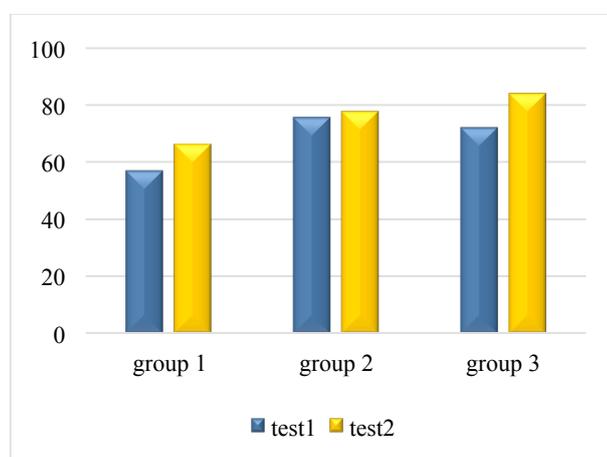


Figure 4. Differences in achievements in geometry at the different time points - Comparison of groups

In order to examine differences between the three groups before and after and the interaction between time and group, a two-way ANOVA analysis was carried out between the groups. It was found that there were no significant differences between the times [$F(1.73)=2.118$; $p=0.150$]. In contrast, differences were found between groups beyond time points [$F(2.73)=3.129$; $p=0.050$], but no interaction was found between time and group [$F(2.73)=0.322$; $p=0.726$].

It was found that experimental group 3 received the highest score on average in the test after the program (84.167). Moreover, the average score of experimental group 3 increased the most (12.084) of all groups between the intermediate test and test on completion of the program.

7.2 Pupils' Attitudes to Geometry

In order to examine pupil's attitudes in the three research groups, the questionnaire about attitudes towards geometry was carried out twice - before and after the interaction. An analysis of the questionnaire data showed no significant difference between the three groups.

7.3 Pupils' beliefs regarding mathematics

In order to examine pupils' beliefs in the three research groups, the beliefs questionnaire was used twice to examine their beliefs about mathematics - before and after the interaction. An analysis of the questionnaire data showed significance, but the change was similar in all three groups.

8. Conclusion

This research examined the effect of the LSP intervention program on pupils' achievements in geometry and their attitudes towards mathematics and geometry. The research hypothesized that group 3 will have the highest achievements in a geometry test after the program. In addition, experimental group 3 would show the greatest improvement between the first and second geometry tests. Moreover, a difference would be found between the groups with regard to problem solving skills and the last assumption was that group 3 would reveal more positive attitudes towards mathematics and geometry.

Using analysis tools it was found that control group 2, whose initial data was highest of the three groups in the Raven test, van Hiele Questionnaire and test to examine their level in geometry. That is to say that is possible to conclude from this that control group 2 had the highest level of geometry prior to introducing the geometry learning.

At the end of teaching geometry and the LSP, according to data analysis, it was found that all three groups had improved their results in the second geometry test in contrast to the first. In addition, it was found that experimental group 3 received the highest average mark of the three groups in the final test (84.167). Moreover, the average mark of group 3 increased the most (12.084) of the three groups between the first test and second test. Adding it to this the finding that improvements were seen in the mean scores in the experimental group 3 at the second van Hiele test. It can be said that it is possible to conclude that the group that underwent the LSP (experimental group 3) on average improved its achievements the most, this despite the fact that in the starting data the control group 2 had the highest average level of knowledge, understanding, analysis and synthesis and as well as in the Raven test and van Hiele Questionnaire. As such, it is possible to say that the first and second research hypotheses were confirmed. Furthermore, it is possible to conclude the same with regard to problem solving skills in geometry. According to data analysis of the intermediate and final tests in geometry it was found that the greatest average improvement was in the experimental group 3 that went through the LSP. As such it is possible to say that hypotheses No. 3 was correct according to data analysis. These findings support the various theories presented in this review, such as Hoffer (1981) who spoke about the combination of skills (visual, verbal, logical etc.) when teaching geometry, will contribute to increasing pupils' interest in the subjects and understanding of learned material, or Galperin's theory (1992b), which argued for the importance of conceptualizing mental processes and abilities whose origins are in meaningful, joint activities, providing cognitive tools and directions as a factor of pupils' development and learning.

With regard to hypothesis No. 4, data analysis showed there was no significant between the groups with regard to capability, intrinsic motivation or self-direction. In addition, there was no difference between the groups in the tests prior to and after conducting the program with regard to beliefs about mathematics, learning mathematics and capability. It is possible that this data emerges for a number of

reasons. It is possible that following the relatively short amount of time between conducting the program and carrying out the research constituted a crucial factor in the significance of the results, the process of internalization and learning is a long process, especially when speaking about beliefs and attitudes, for them to be significantly assimilated. This is supported by Vygotsky's (1978) theory that argues that developmental and learning processes are not the same. The developmental process follows the learning process. The learning process takes place in the zone of proximal development area and the learning process becomes a development process (Miller, 2011). Further support can be seen in Galperin's (1992b) theory that conceptualizes internalization action as a transformative process of individuals' external activities to another mental form of the same external activity, and as a consequence to assimilate new knowledge and skills. That is to say that the time factor should also be emphasized in order to examine beliefs and attitudes expressed by children's developmental processes.

In conclusion, the article chose to bring quantitative examination to the research hypotheses, but one must remember that mixed method research was chosen, out of an understanding that variables such as beliefs and attitudes towards mathematics and geometry can be examined more accurately and profoundly in qualitative research, using interviews. In addition, qualitative research methods can glean a greater and more relevant amount of information about pupils' cognitive thinking processes in solving mathematical problems using the 'thinking person' strategy.

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