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**A BAYESIAN DATA SCIENCE APPROACH TO A MULTILEVEL
PROBLEM IN HRM**

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Abstract

Data arising in Human Resource Management research usually comes from variables measured at different levels of analysis. These data presents an inherent multilevel structure. Single level models, such as simple or multiple regression, are not suitable for efficient modeling. Hierarchical Linear Models are more appropriate tools, especially if estimated with a Bayesian statistical approach. Three different models have been fitted to a typical multilevel HR data set and compared in a Bayesian setting: a single-level linear model, a two-levels linear model with random intercept and slope, and a cross-level interaction model. The three models have been compared on the basis of the difference in expected log predictive density. The cross-level interaction model was the best one. Reasons for considering the Bayesian framework as the most natural environment for analyzing multilevel data generated in HRM research are outlined. Some advantages, peculiarities, and capabilities of modern software packages designed for Bayesian statistical analysis are also outlined.

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1. Introduction

Sources of data in Human Resource Management (HRM) are usually placed at several different levels: firm, organization, unit, team, and individual. Employee's behaviour may be affected by both personal attitudes and organization variables. This is why research in this area asks for parametric models with more than one level (Ostroff & Bowen, 2000; Shen, Messersmith, & Jiang, 2018). Due to the multilevel nature of data structure, single level linear modelling (simple or multiple classical regression) is not suited for exploration, modelling, and prediction of data arising in HRM (Aguinis, Gottfredson, & Culpepper, 2013; Shen et al., 2018; Shen, 2016). When conducting research that includes variables measured at different levels of analysis, researchers explicitly recognize that lower-level entities, such as individuals, are nested within higher-level collectives such as teams (Aguinis et al., 2013). The multilevel nature of data requires dependence among observations to be considered both conceptually and analytically (Snijders & Bosker, 2012). A higher-level variable may covary with relevant lower-level outcomes variables, and entities within collectives may be more similar regarding certain variables compared to entities across collectives (Bliese & Hanges, 2004). It may also happen that the nature of the relationship between lower-level variables depends on higher-level ones, the so-called *cross-level interaction effect* (Aguinis et al., 2013). A Hierarchical Linear Model (HLM) also called Multilevel Model or Mixed Model, explains variance among variables at more than one level and is usually better fitted to data arising in HRM. HLM allows researchers to understand whether relationships between lower-level variables, such as individual job satisfaction and performance, firm capabilities and performance, change as a function of higher-order moderator variables, such as leadership climate or marked-based conditions (Aguinis et al., 2013; Shen et al., 2018).

HLM in social and behavioural sciences, as well as in HRM, are more often estimated in the frequentist statistical framework (Aguinis et al., 2013; Cohen, Cohen, West, & Aiken, 2003; Muth, Oravec, & Gabry, 2018). However a Bayesian statistical approach to HLM fitting, besides being the most natural, offers several advantages, in terms of interpretability of estimates and flexibility to increasingly complex models (Muth et al., 2018; Korner-Nievergelt et al., 2015; McElreath, 2016). Until fairly recently, due to computational limitations, practical Bayesian statistical analysis was limited to few applications. Markov-Chain Monte Carlo (MCMC) methods, a suite of random-sampling simulation methods originally formulated in the 1950s in statistical physics, has greatly expanded the application of Bayesian methods to applied data analysis (Fox & Weisberg, 2019; Gelman et al., 2013; Kruschke, 2015). Bayesian modelling has become increasingly accessible and efficient thanks to advances in statistical software and generic estimation engines such as Stan (Stan Development Team, 2017; Carpenter et al., 2017; Kruschke, 2015). Stan implements a species of MCMC methods, called Hamiltonian Monte Carlo, which provides advantages of robustness and efficiency relative to older MCMC methods such as the Metropolis-Hastings algorithm and the Gibbs sampler (Fox & Weisberg, 2019). Several modern Stan-based R packages make some Stan's functionality available using programming techniques familiar to R users. In this paper we have taken advantage of using the `rstanarm` R package (Stan Development Team, 2017; Carpenter et al., 2017) that make Bayesian model specification more easy and succinct by miming the syntax of several R's functions very popular in the frequentist statistical community.

It is well known that, among operative functions of modern HRM, improvement of employees' relations, team building and leadership management, play a crucial role. As a result of the widespread move to team-based organizations in industry, managers are often asked to lead and motivate both individuals and teams as a whole. When building and fitting statistical models, researchers in this field should consider the dynamic interplay between the individuals within a team and the team as a whole (Chen, Kirkman, Kanfer, Allen, & Rosen, 2007).

In this paper we have considered a typical HR data set concerning individuals grouped in several teams. The data set presents a natural nested (hierarchical) structure and was originated and analyzed in Aguinis et al. (2013). The data set considered in that paper, was patterned after Chen et al. (2007). Among other concepts, several multilevel models were fitted to the data and compared, but in a frequentist statistical setting. However multilevel models naturally fit into the Bayesian paradigm (Kruschke, 2015; Shen et al., 2018; Gelman et al., 2013). This is the reason, in addition to various advantages, for which we have analysed the same data in a Bayesian statistical framework. One single-level linear model, one two-level hierarchical linear model and one cross-interaction two-level linear model have been fitted and compared. General advantages of a modern Bayesian statistical approach, in terms of interpretability of the results, and of modern analytical and graphical tools have been outlined. For the computational aspect of this paper we have taken advantage of the Stan software (Stan Development Team, 2017) and of the R statistical software and some of its packages (R Core Team, 2018).

2. Literature Review and Theoretical Framework

In this section we presents a review of literature on multilevel modelling in HRM research, on modern Bayesian statistical analysis, and on software availability.

2.1. Literature review

For a state of art, wide and exhaustive introduction to multilevel thinking and modelling in HRM research, we refer to Bliese (2002), Renkema, Meijerink and Bondarouk (2017), Shen et al. (2018), Shen (2016), Aguinis et al. (2013), and the references therein. Gelman and Hill (2007) is a good, perhaps dated from a computational point of view, introduction to general Bayesian statistical methods. Gelman et al. (2013) is a quite complete, but mathematically and technically demanding, presentation of modern Bayesian methods with an appendix on using Stan and R. McElreath (2016) is a gentle introduction to Bayesian methods, including multilevel linear models, with R and Stan. Fox and Weisberg (2019) is a good reference for applied regression with R, including multilevel (mixed) models, with a freely available web appendix on Bayesian estimation with Stan. For the Stan program, the state of art in Bayesian statistical software, we manly refer to Carpenter et al. (2017). Gabry and Goodrich (2018) present the rstanarm package, a Stan based R package. Gabry, Mahr, Bürkner, Modrák, and Barret (2018) is the reference for the bayesplot package for plotting Bayesian models. Muth et al. (2018) is an excellent introduction to Bayesian hierarchical linear models fitting with rstanarm package and the user-friendly exploration of results with the shinystan package. References therein can guide the interested readers towards a deeper knowledge of the subject.

2.2. The Bayesian approach to model fitting with *Stan* and *rstanarm*.

The statistical models presented later in this paper can all be estimated and fitted to data with a frequentist approach. However when data presenting a multilevel structure asks for hierarchical modelling, the Bayesian statistical approach is the most suitable one. The starting point of Bayesian statistics is Bayes' theorem. In a regression model setting its general form states that the posterior probability distribution $p(\theta|y, x)$ of parameters θ given data y and x is proportional to the product of the likelihood function, $p(y|\theta, x)$, of the observed response data y given the parameters θ and the regressor data x , and the prior distribution $p(\theta)$ of the parameters:

$$p(\theta|y, x) \propto p(y|\theta, x)p(\theta).$$

Here we have assumed that, as it is the case in standard regression setting, x and θ are independent, so that the prior $p(\theta)$ doesn't depend on the regressor x .

The most compelling applications of Bayesian methods typically aren't to standard regression models, such as the normal linear model or generalized linear models, but to problem where we want to specify a customized probability model for the data such as a Hierarchical Linear Model, in short HLM (Fox & Weisberg, 2019; Muth et al., 2018). HLM is an extension of the classical linear model that is well suited for data like ours that present a nested structure where individuals are grouped in teams. Relationships are expected to be similar in the same group and groups are not generally exchangeable (Gelman & Hill, 2007; Hox, Moerbeek, & van de Schoot, 2018). Modeling in the Bayesian statistical framework is mainly composed of four key steps (Muth et al., 2018): data model and prior specification, parameters estimation, sampling quality and model fit checking, results summarization and interpretation. Here, because of space limitations, we will not deeply explore all the four steps. The Stan estimation engine has been used to fit the statistical models described in the next sections via the *rstanarm* package. We used the default weakly informative priors of *rstanarm*. The *rstanarm* package allows the user to easily specify the models considered (Carpenter et al., 2017). Several fundamental tools for MCMC diagnostic are also implemented in the package as well as useful numerical device for results summarization. Graphical representation tools used in this paper are provided by the *bayesplot* package (Gabry et al., 2018).

We encourage the reader to further explore the literature on Bayesian methods, for example via Gelman and Hill (2007), Gelman et al. (2013), Kruschke (2015), McElreath (2016).

3. Research Method

In this section we present the data and the statistical models to be fitted to them.

3.1. The data

Our data set is part of a publicly available larger data set originated by Aguinis et al., (2013) and patterned after a study by Chen et al. (2007), composed of individual observations of several variables on 630 employee. We have selected 4 columns from the original data set, named as: quality of *leader-member exchange (LMX)*, *individual empowerment (IE)*, *leadership climate (LC)*, and *Team*. So our data set is composed of 630 rows (observations) and 4 columns (variables). The 630 employees are nested in

105 teams, identified by the *Team* variable, each composed of 6 individuals. The *leadership climate* is a *Team* dependent variable. The goals of the previously cited studies were to investigate whether the quality of *leader-member exchange (LMX)* predicts *individual empowerment (IE)* and the role of the team dependent *leadership climate (LC)*. Chen et al.'s model predicted that individuals who report a better relationship with their leader (i.e., a higher *LMX* value) has the autonomy and capability to perform meaningful works that positively affect their organization (i.e., they feel more empowered). The model assumed also that the team-level variable *LC* would affect the individual-level empowerment *IE* positively. It was also supposed that the relationship between *LMX* and *IE* would be stronger for teams with higher *LC*. See Aguinis et al. (2013) for more information on the data.

3.2. The statistical models

The first model considered in this paper is a simple linear model explaining the relationship between the level-1 variables *individual empowerment (IE)* and *leader-member exchange (LMX)*:

$$IE_i = b_0 + b_1 LMX_i + \varepsilon_i, \quad \varepsilon_i \sim Normal(0, s^2).$$

We will refer to this model as *modell*. This model could be considered a conceptually wrong one, because it ignores the nested (hierarchical) structure of our data. However we will fit it to data to derive an overall relationship between *IE* and *LMX*. We will later compare *modell* to each team-specific fitted regression line considered in *model2* (see Figure 01), as defined below.

The second considered model, referred as *model2*, is a *hierarchical two-levels model*, with random intercept and random slope:

$$IE_{ij} = b_0 + b_{0j} + (b_1 + b_{1j}) LMX_{ij} + \varepsilon_{ij},$$

$$\begin{pmatrix} b_{0j} \\ b_{1j} \end{pmatrix} \sim Normal \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} s_0^2 & r s_0 s_1 \\ r s_0 s_1 & s_1^2 \end{pmatrix} \right), \quad \varepsilon_{ij} \sim Normal(0, s^2).$$

This model improves *modell* by taking into account the nested structure of the data: employees are grouped in teams. It is expected that people in the same team share similar *IE-LMX* relationship. The strength of this relationship is supposed to vary among teams. In *model2*, parameters b_0 and b_1 are the overall intercept and slope parameters, while $b_0 + b_{0j}$ and $b_1 + b_{1j}$ are the intercept and slope parameters for the j th team. The error ε_{ij} reflect individual-level variation within each team, and its variance s^2 quantifies the within-team variation. The variance of b_{0j} , denoted by s_0^2 , quantifies the degree of heterogeneity in intercept across teams. The variance of b_{1j} , denoted by s_1^2 , quantifies the degree of heterogeneity in slope across teams. The parameter r represents the correlation between intercepts and slopes and it should be interpreted in the following way: a positive value for r means that teams with stronger *IE-LMX* relationship tend to have higher *IE* level (Aguinis et al., 2013).

The third model, *model3*, is a *cross-level interaction hierarchical model*, with random intercept and random slope:

$$IE_{ij} = b_0 + b_{0j} + (b_1 + b_{1j}) LMX_{ij} + b_2 LC_j + b_3 LMX_{ij} \times LC_j + \varepsilon_{ij},$$

$$\begin{pmatrix} b_{0j} \\ b_{1j} \end{pmatrix} \sim Normal \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} s_0^2 & r s_0 s_1 \\ r s_0 s_1 & s_1^2 \end{pmatrix} \right), \quad \varepsilon_{ij} \sim Normal(0, \sigma^2).$$

This is the model considered in *Step 4* of Aguinis et al., 2013. Note that *model3* presents the same hierarchical structure of *model2*, but it is also hypothesized, as in Chen et al. (2007), that the team-level variable *LC* would affect individual-level *IE* and the relationship between *IE* and *LMX*.

For more information about models as *model2* and *model3*, readers may refer to Aguinis et al. (2013), Hox et al., (2018), Snijders & Bosker (2012).

3.3. The software

We have intensively used the Stan program (Carpenter et al., 2017) via the rstanarm package (Gabry & Goodrich, 2018), which provides an R software interface to Stan programming. The bayesplot R package (Gabry et al., 2018) has complemented the rstanarm tools providing an extensive library of function for plotting posterior draws, visual MCMC diagnostic, as well as graphical posterior predictive checking.

4. Findings

In this section we present part of the outputs of the Bayesian analysis aimed to fit models to data. Results for different models are in different subsections.

4.1. Results for *modell* fitting.

The estimated posterior mean, standard deviation and 95% Credible Intervals (CI's) for the intercept *b0*, the slope *b1* and the residual standard error *s* for *modell* are reported on Table 01.

Table 01. Posterior mean, standard deviation (st.dev.), and 95% CI's (2.5%, 97.5%) for parameters of *modell*

	mean	st.dev.	2.5%	97%
b0	5.720	0.0332	5.654	5.784
b1	0.279	0.0258	0.227	0.328
s	0.828	0.0234	0.785	0.875

Table 02. Posterior mean, standard deviation (st.dev.), and 95% CI's (2.5%, 97.5%) for relevant parameters of *model2*

	mean	st.dev.	2.5%	97.5%
b0	5.7193	0.0443	5.63184	5.8072
b1	0.2708	0.0282	0.21462	0.3245
s	0.7231	0.0243	0.67761	0.7735
s0	0.1264	0.0300	0.07593	0.1922
r	0.0144	0.0128	-0.01009	0.0401
s1	0.0265	0.0122	0.00656	0.0533

4.2. Results for *model2* fitting.

Results for *model2* has 213 more estimated parameters than results for *modell*. This is because *model2* estimates intercept and slope for each of the 105 teams, and it estimates also the level-2 spread in terms of st. dev.s s_0 , s_1 , and correlation r . In Table 02 are reported the estimated mean, standard deviation and 95% C.I. for the overall population intercept b_0 , the overall population slope b_1 , the level-1 residual standard error s , the between-team intercept standard deviation s_0 , the between-team slope standard deviation s_1 , the between-team intercept-slope correlation r . For space saving, Table 02 is truncated to exclude the 210 estimated intercept and slope increments for all teams. One of the best features of Bayesian estimation is that it returns (posterior) samples for estimated parameters, from which we can derive estimated probability density functions.

In Figure 01, such estimated density are represented for relevant parameters of *model2*, with 80% CI's. Note that all CI's don't contain 0. This can be interpreted in a Bayesian framework as evidence that all parameters should be considered significantly different from 0.

In Figure 02 a graphical comparison of *modell* and *model2* is represented for the first 4 teams. Dots are the observed data. The dashed line is the estimated posterior regression line from *modell*. The dark lines are the estimated posterior regression lines from *model2*. The light lines represent uncertainty in posterior intercept and slope parameters for each team.

Table 03. Posterior mean, standard deviation (st.dev.), and 95% CI's (2.5%, 97.5%) for relevant parameters of *model2*

	mean	st.dev.	2.5%	97.5%
b_0	5.7193	0.0443	5.63184	5.8072
b_1	0.2708	0.0282	0.21462	0.3245
s	0.7231	0.0243	0.67761	0.7735
s_0	0.1264	0.0300	0.07593	0.1922
r	0.0144	0.0128	-0.01009	0.0401
s_1	0.0265	0.0122	0.00656	0.0533

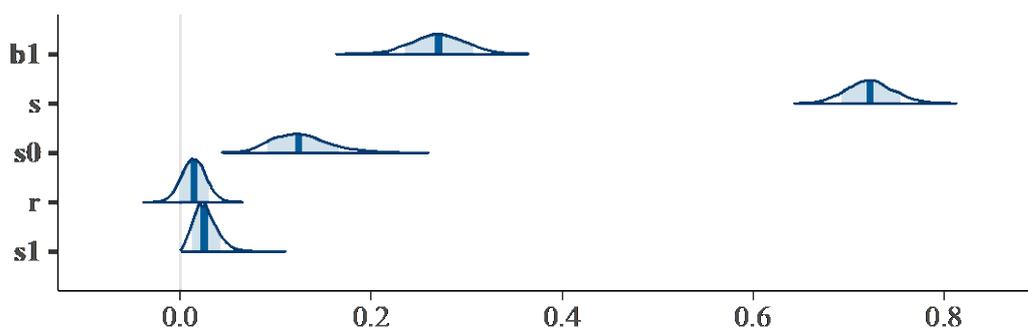


Figure 01. *model2*: Posterior medians with 80% and 90% credible intervals for relevant parameters

4.3. Results for *model3* fitting.

Results for *model3* comprise 2 more estimated parameters than results for *model2*. This is because *model3* estimates also coefficients for *leadership climate* and for *leader-member exchange:leadership climate* interaction. The estimated mean, standard deviation and 95% C.I. for the relevant parameters of *model3* are reported in Table 03. For space saving, Table 03 is truncated to exclude the 210 estimated intercepts and slopes for all teams. In Figure 03 are represented the posterior medians and 80% and 90% credible intervals for coefficients of *LMX*, *LC*, and *LMX:LC* interaction. All intervals don't contain 0. This means that *model3* should be considered a better model than *model2*.

Results of a more analytic model comparison are presented in the next subsection.

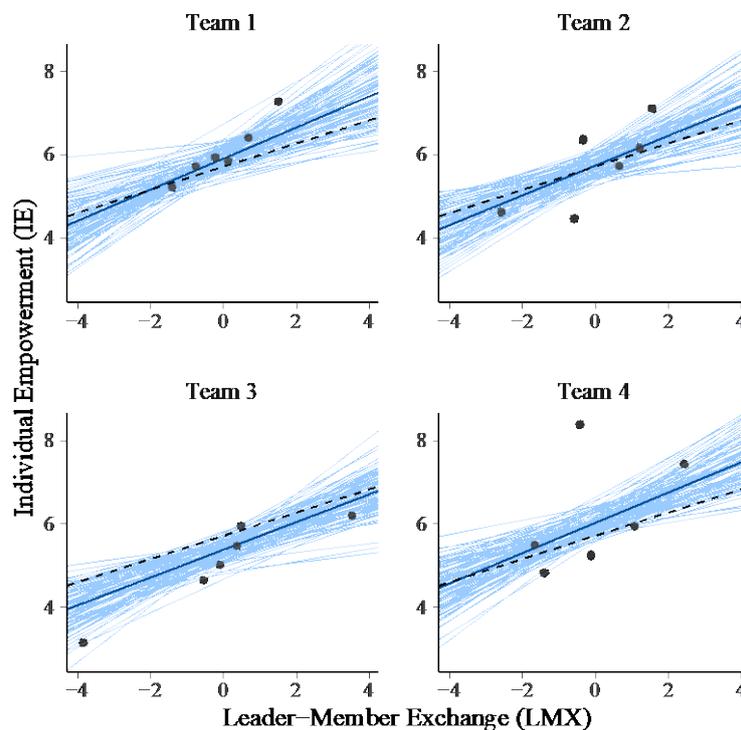


Figure 02. Graphical comparison of *model1* and *model2* for the first 4 teams. Observed data (dots); *model1* posterior mean regression line (dashed line); *model2* posterior mean regression lines (dark lines), and uncertainty (light lines)

Table 04. Posterior mean, standard deviation (st.dev.), and 95% CI's (2.5%, 97.5%) for relevant parameters of *model3*

	Mean	st.dev.	2.5%	97.5%
b0	5.72004	0.0379	5.64564	5.7931
b1	0.26970	0.0272	0.21516	0.3224
b2	0.35052	0.0542	0.24199	0.4572
b3	0.10507	0.0374	0.03101	0.1788
s	0.72720	0.0262	0.67819	0.7797
s0	0.06366	0.0233	0.02046	0.1131
r	-0.00336	0.0100	-0.02392	0.0152
s1	0.01934	0.0112	0.00195	0.0452

4.4. Model comparison.

We have evaluated our models with the leave-one-out (loo) cross-validation and compared them on the basis of the difference in expected log predictive density (*elpd-diff*). When comparing *modell* and *model2*, we have found *elpd-diff*=35.2 with an estimated standard error *st.err.*=7.3. The positive difference in *elpd* (more than twice the estimated standard error) indicates that *model2* is expected to have a better predictive performance than *modell*. When comparing *model2* and *model3*, we have found *elpd-diff*=9.4 with an estimated standard error *st.err.*=5.0. It follows that *model3* should be preferred to *model2* for the predictive performance. For computation we have used the *loo* R package. For more information on the *loo* package and leave-one-out cross-validation and model comparison please refer to Vehtari, Gelman and Gabry (2017).

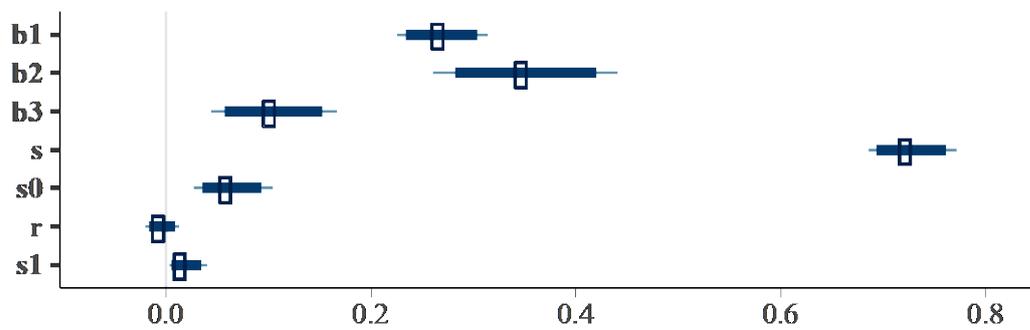


Figure 03. Estimated posterior medians with 80% and 90% credible intervals for relevant parameters of *model3*

5. Conclusion and Discussions

Research in HRM is inherently a multilevel field of study as data originated in this field usually has an intrinsic hierarchical structure. Although until a few years ago HRM research has been conducted only at the single level of analysis, single level statistical tools, such as linear model, are not ideal. Multilevel modelling was first used in education and marketing research (Shen et al., 2018). Very recent trends have witnessed a growing use of hierarchical linear model (HLM) in HR literature, but estimation efforts of these models are more often faced with a frequentist statistical approach. The Bayesian statistical approach is actually ideally suited for constructing hierarchical models, and it is therefore very useful for analysing data structures with multiple levels, such as data from individuals who are members of groups which in turn are in higher-level organizations (Kruschke, 2015; Kruschke & Vanpaemel, 2015; Gelman et al., 2013).

In this study we have fitted in a Bayesian statistical framework three different models to a data set originated and analyzed in Aguinis et al. (2013) with a frequentist approach. The goal of that paper was to investigate whether the quality of *leader-member exchange (LMX)* predicts *individual empowerment (IE)* and the role of the team dependent *leadership climate (LC)*. In this paper we have considered similar multilevel models, but our goal was to show the advantage of a Bayesian estimation approach. The first model considered here, *modell*, is a single level simple linear model to simply predict *IE* from *LMX*. We

have then casted the problem in a hierarchical, or two-levels, modeling framework. The second model, *model2*, include random intercept and slope by nesting *IE* and *LMX* data in the 105 teams into which the employees were grouped. Simultaneously measuring team-level and population-level trends, *model2* improved estimation accuracy. The last considered model, *model3*, was a cross-level interaction hierarchical model, with random intercept and random slope. It was proved that the team-level variable *LC* affect the individual-level empowerment *IE* positively, and that the relationship between *LMX* and *IE* is stronger for teams with higher *LC*. Some of the results were already obtained in Aguinis et al. (2013) in a frequentist paradigm, but we have shown the richness of the Bayesian statistical analysis outputs and some graphical capabilities of modern paraphernalia available to Bayesian analysts via several R packages.

Although the majority of HRM research has historically been conducted at the single level of analysis (Shen, 2016), today it is well known that multilevel modelling significantly advances HRM research by more accurately predicting HRM effects and estimating complex HRM models (Shen et al., 2018). In this paper we have shown, with a concrete example, how modern computational, diagnostic and representation tools have made the Bayesian statistical approach the most convenient as well as the most natural paradigm for analysis and modelling of multilevel data arising in HRM research.

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